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# Optimal Functionally Graded Metallic Foam Thermal Insulation

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**Optimum density profiles that minimize heat transmission through a metal foam thermal insulation under one-dimensional steady-state conditions are investigated. The effective thermal conductivity of the foam is derived in terms of cell parameters and the temperature. Maximizing the temperature at the outside wall of the insulation minimizes the heat conduction through the insulation because this maximizes the radiated heat. An optimality condition is derived, and the optimization problem is reduced to that of an ordinary, but a nonlinear differential equation, which is solved numerically. The optimum density variation through the thickness of the insulation for a given incident heat flux and the transmitted heat are presented for graded and uniform foams with open and closed cells. For open-cell foams, functional grading of the foam density can reduce the heat transfer through the foam for given thickness. Conversely, for a specified amount of heat transmission through the foam, the functionally graded foam insulation can be made thinner than uniform density foam insulation.**

## Nomenclature

| | | |
|---|---|---|
| $a$ | = | foam cell size in plane transverse to direction of heat transfer |
| $a_i$ | = | polynomial coefficients used to approximate optimum foam conductivity as function of temperature |
| $b$ | = | foam cell size in the direction of heat transfer |
| $d_g$ | = | gas collision diameter |
| $d_s$ | = | diameter of the struts of open-cell foam |
| $h$ | = | thickness of foam insulation |
| $K_B$ | = | Boltzmann's constant |
| $k_e$ | = | effective heat transfer coefficient for the foam due to combined gas and solid |
| $k_g$ | = | effective conduction heat transfer coefficient of gas medium inside the foam cells |
| $k_g^*$ | = | thermal conductivity of gas at one atmosphere pressure |
| $k_m^*$ | = | thermal conductivity of the material used for the foam |
| $k_r$ | = | effective heat transfer coefficient for radiation transfer inside the foam cells |
| $k_s$ | = | effective heat transfer coefficient for conduction through solid medium |
| $k^*(T)$ | = | optimum heat transfer coefficient for a given temperature $T$ |
| $k(\rho, T)$ | = | effective conductivity of foam (function of density and temperature) |
| $L$ | = | Lagrangian function for the optimization problem |
| $l_c$ | = | characteristic length for pores/cells in porous media/foams |
| $P$ | = | pressure of gas medium inside the foam |
| $Pr$ | = | prandtl number |
| $Q_0$ | = | incident heat flux on the hot/outer surface of the foam |
| $\bar{Q}_0$ | = | heat flux transmitted through the foam insulation |
| $T$ | = | temperature, K |
| $T_h$ | = | temperature on hot/outer surface |
| $T_0$ | = | temperature on cool/inner surface |
| $V_f$ | = | volume fraction of material in foams |
| $V_f^*$ | = | optimum volume fraction of foams that minimizes heat transfer |
| $x$ | = | coordinate direction along the thickness of the foam insulation |
| $\alpha$ | = | accommodation coefficient |
| $\gamma$ | = | specific heat ratio |
| $\varepsilon$ | = | surface emissivity |
| $\lambda_m$ | = | mean free path length of gas molecules |
| $\rho$ | = | apparent density of foam |
| $\rho_m^*$ | = | density of material used in foam |
| $\sigma$ | = | Stefan–Boltzmann constant |

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## Introduction

METAL foams[1,2] are being investigated for use in multifunctional structures for reusable launch vehicles. Such multifunctional structures would insulate the vehicle interior from aerodynamic heating, as well as carry primary vehicle loads. Varying the density, geometry, and/or material composition from point to point within the foam can produce functionally graded materials (FGM) that may function more efficiently. There is an increasing body of theory and experiments on such materials; however, there is still a great deal of uncertainty as to what can be manufactured in terms of density and material architecture and their variations from point to point. There is similar uncertainty in the structural and thermal properties of the resulting FGM.

To guide development and testing of new FGMs for thermal protection systems, it is important to identify situations where FGMs could contribute most in terms of efficiency gains. This paper seeks density profiles that maximize thermal insulation efficiency based on currently available models for effective thermal conductivity of metallic foams. In the meantime, new micromechanical models need to be developed for functionally graded thermal protection material systems.

In this paper, an optimality criterion is developed for minimizing heat conduction through metal foam with variable cell size through its thickness. Analytical expressions are developed to represent three modes of heat transfer through the foam: solid conduction, gas conduction, and radiation. The resulting effective thermal conductivity of the foam is a function of temperature, pressure, properties of the foam material, and the foam geometry. The optimization problem is to determine the density distribution that minimizes the one-dimensional steady-state heat conduction through the thickness of titanium foam. The optimality condition is used to obtain the optimum density profile. The functionally graded foam and uniform density foam are compared to illustrate performance payoffs provided by optimization of graded foam properties. Derivations and results are presented in the body of the paper for open-cell foams and in the Appendix for closed-cell foams.

## Effective Thermal Conductivity of Metal Foam

An analytical model of the heat transfer through the foam is required to investigate the optimum design of functionally graded metallic foam insulations. Here we investigate open-cell foam with spatial variation in its cell size. Heat transfer through porous metal foams involves a number of heat transfer modes. Equations are developed for gas conduction, metal conduction, and radiation. Forced convection and natural convection are also possible modes of heat transfer through metal foams. Designing the insulating system to avoid airflow paths can eliminate forced convection. Natural convection in metal foams is a complex function of the air density and foam geometry and orientation with respect to gravity or inertial forces. Much of the heat transfer during atmospheric entry occurs when air density is low, thereby inhibiting natural convection. Small pores, which tend to reduce the other modes of heat transfer, also inhibit natural convection within the foam. Foams could be oriented at almost any angle with respect to gravity and inertial forces if used over a significant portion of a vehicles surface. Therefore, convection is neglected for the current study.

The heat transfer equations presented here are based on an idealized model of metal foam, where the foam is modeled as a periodic rectangular cellular material[3] with a repeating volume element, as shown in Fig. 1 for the open-cell foams. More detailed complex models[4,5] of open-cells foams that accurately represent the structure of open-cells foams have been proposed more in recent literature. However, such models have not been experimentally verified at high temperatures. Models similar to those used in this paper were also compared by Sullins and Daryabeigi[6] to experimental results and shown to be valid. The simplified model used here, although quite simplistic, has been shown to describe the effect of pore size and material density on the heat transfer modes. The model used here was found to exhibit the same dependence of the heat transfer on density and pore cell size as reported by Paek et al.[7]

The dimensions of the cell are given by $a$ in the plane transverse to the direction of heat transfer and $b$ in the direction of heat transfer. The diameter of the struts of open-cell foam is given by $d_s$. The volume fraction $V_f$ of metal in the open-cell foam, referred to as solidity, is given by the expression

$$V_f = \frac{\text{volume in struts}}{\text{total volume}} = \frac{(8a+4b)\left(\pi d_s^2/4\right)/4}{a^2 b} = \frac{\pi}{4}\left(2\frac{a}{b}+1\right)\left(\frac{d_s}{a}\right)^2 \tag{1}$$

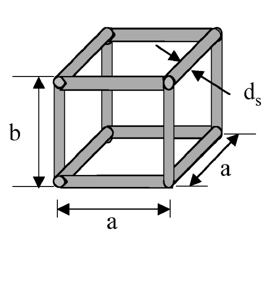


**Fig. 1 Rectangular representative volume element used for heat transfer model in open-cell foam.**

For a perfect cubic cell, the expression for solidity reduces to the following equation:

$$V_f = \tfrac{3}{4}\pi (d_s/a)^2 \tag{2}$$

The effective heat transfer coefficient is calculated as the linear combination of three components: conduction through the gas, conduction through the metal, and radiation. The expressions used for the different heat transfer modes are briefly described in the following sections.

### Gas Conduction

Air conduction is modeled in the voids in the metallic foam. Away from any solid boundaries, the thermal conductivity of a gas is independent of pressure.[8] However, gas molecules interact with solid surfaces in such a way that the thermal conductivity of a gas enclosed in a small cavity does vary with pressure. The pressure-dependent thermal conductivity for air inside the foam, $k_g$, is calculated using the following equation[9]:

$$k_g = (1-V_f)\left\{k_g^* \Bigg/ \left[1 + 4\frac{(2-\alpha)}{\alpha}\frac{\gamma}{(\gamma+1)}\frac{1}{Pr}\frac{\lambda_m}{l_c}\right]\right\} \tag{3}$$

where $V_f$ is the volume fraction of the solid material in the foam, $k_g^*$ is the air conductivity at 1-atm pressure, $\alpha$ is the accommodation coefficient (a measure of the average efficiency of the energy exchange between gas molecules and solid surfaces, assumed equal to 1.0), $\gamma$ is the specific heat ratio (assumed equal to 1.4 for air), $\lambda_m$ is the mean free path length, and $l_c$ is the characteristic length of the pores in the foam. The factor $(1-V_f)$ is introduced in the expression to account for the reduced volume fraction of gas in porous foam. The temperature dependence of the air conductivity and the Prandtl number are calculated using the expressions (see Ref. 10)

$$k_g^* = 3.954 \times 10^{-3} + 7.7207 \times 10^{-5} T - 1.6082 \times 10^{-8} T^2 \tag{4}$$

$$Pr = 0.7086 - 3.7245 \times 10^{-6} T + 2.2556 \times 10^{-8} T^2 \tag{5}$$

where $T$ is absolute temperature (degrees Kelvin). The mean free path length $\lambda_m$, which represents the mean distance traveled by gas molecules between collisions with other molecules, is a function of temperature $T$ and pressure $P$ and is given by the expression

$$\lambda_m = \frac{K_B T}{\sqrt{2}\pi d_g^2 P} \tag{6}$$

where $K_B$ is the Boltzmann constant (in joules per degrees Kelvin) and $d_g = 3.65009 \times 10^{-10}$ is the gas collision diameter in meters.

All of the preceding quantities are readily apparent, except for the characteristic length. For closed-cell foams, an obvious choice for characteristic length is the cell size in the direction of heat transfer. For open-cell foams, the choice of characteristic length is not as straightforward. In this study, the characteristic length for open-cell foams was calculated using the following equation derived for fibrous insulation[11]:

$$l_c = (\pi/4)(d_s/V_f) \tag{7}$$

### Solid Conduction

Solid conduction through the foam can be complicated due to the geometry of the pores. A comprehensive study of heat transfer through polymeric foams was published by Glicksman.[3] The following expressions, derived by Glicksman, are more applicable to realistic foam geometries than expressions that could be derived directly from the simplified geometry in Fig. 1.

For open-cell foams, the heat transfer coefficient for conduction heat transfer through the solid phase $k_s$ is given as

$$k_s = \tfrac{1}{3} k_m^* V_f \sqrt{a/b} \tag{8}$$

where $k_m^*$ is the metal conductivity. For titanium foam used in our study, the temperature-dependent metal conductivity (watts per meter per degree Kelvin) is given by[12]

$$k_m^* = 2.7379 + 1.3462 \times 10^{-2} T - 1.7207 \times 10^{-7} T^2 \tag{9}$$

**Radiation**

Radiation through foams is also complicated by foam geometry. Glicksman[3] has addressed the more complex problem of translucent polymeric foams, but also developed expressions for opaque open- and closed-cell foams. Glicksman's equations (for open-cell foams) expressed using the notations of this paper are

$$k_r = 4[\varepsilon/(2-\varepsilon)]\sigma T^3 b \tag{10}$$

where $k_r$ is in watts per meter per degree Kelvin, $\sigma$ is the Stefan–Boltzmann constant with units of $\mathrm{Wm^{-2}K^{-4}}$, $\varepsilon$ is the emittance of the internal surface of the foam (assumed as 0.5 for calculations in this paper).

**Overall Effective Thermal Conductivity**

The overall effective thermal conductivity as function of temperature and volume fraction consists of the contributions of all modes of heat transfer. We assume that the medium is optically thick. The optical thickness is defined as the ratio of the characteristic length to photon mean free path, and for the metal foams used in our study, it is always larger than 10. Consequently, the contributions of the three modes of heat transfer, namely, gas conduction, metal conduction, and radiation, can be linearly combined as

$$k_e = k_g + k_s + k_r \tag{11}$$

The effective thermal conductivity at 1 atm pressure, for the range of temperatures and densities chosen in our study, is shown in Figs. 2 and 3. The variation in density was achieved by changing the cell size with the strut diameter kept constant at 0.05 mm (0.002 in.). Fixing the strut diameter leads to large cell sizes for low-density foams. For dense foams, when heat transfer is dominated by conduction, this is of little consequence, and so the effective conductivity is monotonic function of solidity. For low density, however, large cell sizes increase the radiation heat transfer $k_r$ so that at low solidities the effective conductivity reaches a minimum as shown in Fig. 3. The contribution of the radiation component significantly increases with temperature because it is proportional to the third power of temperature. Therefore, the minimum conductivity is reached at higher solidities at high temperatures as shown in Fig. 3. Because the temperature varies through the thickness of the insulation, this indicates that the solidity (volume fraction) distribution can be optimized to minimize the heat transfer through the foam. In the next section, we derive the optimality condition that describes the optimum density distribution for minimal heat transfer.

## Optimality Criterion for Minimum Heat Transfer

We consider an insulation panel (Fig. 4) of thickness $h$ with a given heat flux $Q_0$ on its hot side, $x = h$, and a given temperature $T_0$ on its cool side, $x = 0$. We assume that there is very little heat transfer in the plane of panel, so that the problem can be treated as one dimensional. Furthermore, the transient effects are neglected.

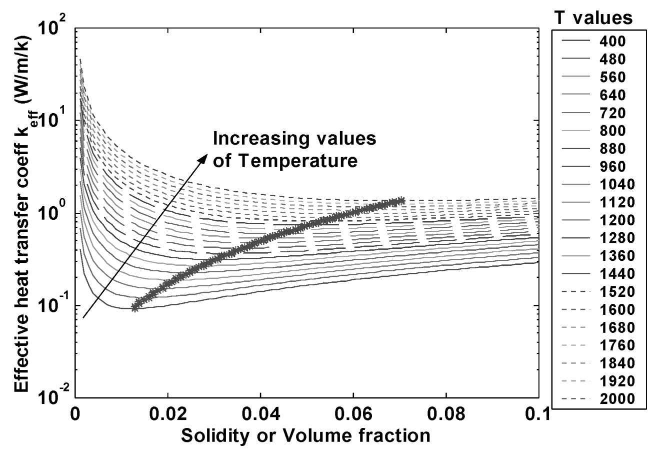


**Fig. 3 Effective conductivity as function of solidity for different temperatures; asterisks denote optimum conductivity points.**

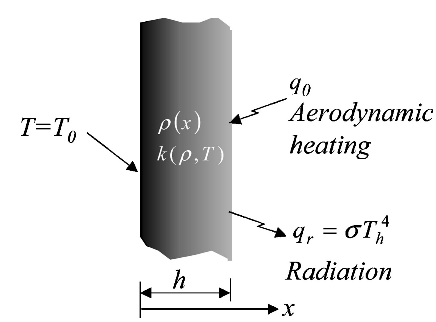


**Fig. 4 Schematic of one-dimensional heat conduction in FGM foam insulation.**

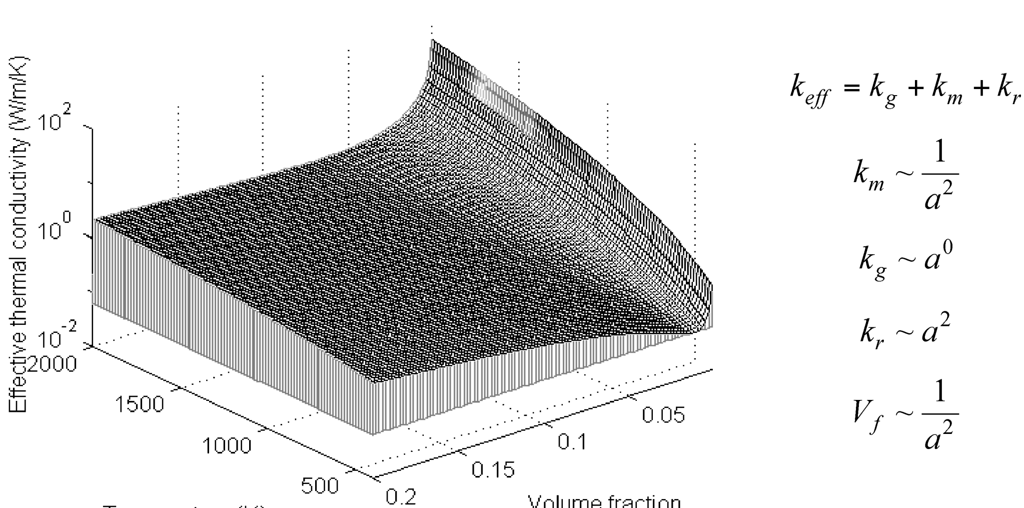


**Fig. 2 Effective thermal conductivity for range of temperatures *T* and volume fraction $V_f$ and dependence of the components of the effective conductivity on the foam pore size *a* obtained for a titanium open-cell foam with fixed strut diameter of 0.05 mm.**

Our objective is to minimize the heat transmitted through the foam insulation for a given thickness. The same can be achieved by maximizing the outside temperature because this increases the radiation at the wall and, hence, minimizes the heat being transmitted through the foam. For one-dimensional heat transfer though the foam with radiation at its outside wall, the steady-state heat transfer equation and boundary conditions are

$$kT' = Q = Q_0 - \sigma T_h^4 \qquad (0 \le x \le h) \tag{12}$$

$$T(0) = T_0 \tag{13}$$

where prime denotes differentiation with respect to $x$. The conductivity is given as $k = k(\rho, T)$. We want to maximize the temperature at $x = h$, subject to Eq. (2) as a constraint, and so we form the Lagrangian

$$L = T_h - \int_0^h \lambda(x)\left[kT' - Q_0 + \sigma T_h^4\right] \mathrm{d}x \tag{14}$$

and take the variation

$$\delta L = \delta T_h - \int_0^h \lambda(x)[\delta k T' + k\delta T' \,\mathrm{d}x + 4\sigma T^3(h)\delta T_h] \,\mathrm{d}x \tag{15}$$

To evaluate this integral we use

$$\delta k = \frac{\partial k}{\partial \rho}\delta\rho + \frac{\partial k}{\partial T}\delta T \tag{16}$$

$$\int_0^h \lambda k \delta T' \,\mathrm{d}x = \lambda k \delta T|_0^h - \int_0^h (\lambda k)'\delta T \,\mathrm{d}x \tag{17}$$

Hence, we obtain

$$\delta L = \left[1 - 4\sigma T_h^3 \int_0^h \lambda \,\mathrm{d}x - \lambda(L)k(L)\right]\delta T_h - \int_0^h T'\lambda \frac{\partial k}{\partial \rho}\delta\rho \,\mathrm{d}x + \int_0^h (\lambda k)'\delta T \,\mathrm{d}x - \int_0^h \lambda \frac{\partial k}{\partial T} T'\delta T \,\mathrm{d}x \tag{18}$$

This gives us the following optimality conditions:

$$T'\lambda \frac{\partial k}{\partial \rho} = 0, \qquad (\lambda k)' - \lambda T' \frac{\partial k}{\partial T} = 0 \tag{19}$$

In the first condition, $T' = 0$ implies uniform temperature that corresponds to no heat transfer. Therefore, the optimality condition becomes

$$\frac{\partial k}{\partial \rho} = 0 \tag{20}$$

This optimality condition corresponds to the expected result that for given $T$ we seek the density that minimizes $k$.

On expanding the terms of the second condition, we have

$$\lambda' k + \lambda\left(\frac{\partial k}{\partial \rho}\rho' + \frac{\partial k}{\partial T}T'\right) - \lambda T' \frac{\partial k}{\partial T} = 0 \tag{21}$$

On substituting Eq. (20) in Eq. (21) and simplifying, we obtain

$$\lambda' k = 0 \qquad \text{or} \qquad \lambda = c(\text{const}) \tag{22}$$

For a fixed value of the strut diameter, the effective conductivity (heat transfer coefficient) is a function of temperature $T$ and volume fraction or solidity $V_f$, that is,

$$k = k(T, V_f) \tag{23}$$

Because the density is proportional to the volume fraction ($\rho = \rho_m^* V_f$), the optimality condition (20) can be rewritten as

$$\frac{\partial k}{\partial V_f} = 0 \tag{24}$$

Substituting the expression for effective conductivity in the optimality condition provides us the values of optimum solidity (or volume fraction $V_f^*$) and heat transfer coefficient $k^*$ as a function of temperature.

## Optimality Criterion for Minimum Thickness

Instead of using the functional grading to reduce the fraction of heat that is transmitted through the wall, it is possible to reduce the thickness for a given value of transmitted heat. To minimize the thickness, it is convenient to write the heat transfer equation as

$$\frac{\mathrm{d}x}{\mathrm{d}T} = \frac{k}{\bar{Q}_0} \tag{25}$$

where

$$\bar{Q}_0 = Q_0 - \sigma T_h^4 \tag{26}$$

On integrating Eq. (24), we obtain

$$h = \int_{T_0}^{T_h} \frac{\mathrm{d}x}{\mathrm{d}T}\,\mathrm{d}T = \int_{T_0}^{T_h} \frac{k}{\bar{Q}_0}\,\mathrm{d}T \tag{27}$$

with

$$k = k(\rho, T) \tag{28}$$

The variation of Eq. (28) is of the form

$$\delta h = \int_{T_0}^{T_h} \left(\frac{\partial k}{\partial \rho}\delta\rho\right)\frac{\mathrm{d}T}{\bar{Q}_0} \tag{29}$$

from which the optimality criterion is once again

$$\frac{\partial k}{\partial \rho} = 0 \tag{30}$$

The discrete form of Eq. (24) given by

$$\Delta x = (k/\bar{Q}_0)\Delta T \tag{31}$$

is used for the numerical evaluation of the FGM thickness. The temperature difference between inside and outside wall ($T_h - T_0$) is divided into small intervals $\Delta T$, and for each interval, the optimum thickness is calculated using

$$\Delta x_i = \left[k^*(T_i)\big/\bar{Q}_0\right]\Delta T \tag{32}$$

where $k^*(T_i)$ is the minimum thermal conductivity corresponding to the average temperature in the interval $T_i$.

## Numerical Evaluation of Designs from Optimality Criterion

The dependence of the effective conductivity or heat transfer coefficient as a function of temperature and solidity is shown in Figs. 2 and 3. Notice that for each temperature there is a unique value of the heat transfer coefficient or solidity at which the optimality condition is satisfied. The values of the conductivity $k^*$ and volume fraction that satisfy the optimality condition were obtained numerically (Fig. 3). The optimum heat transfer coefficient $k^*$ and solidity $V_f^*$ are then fitted with using a quadratic polynomials to obtain algebraic expressions that describe their dependence on temperature approximated by a polynomial expression (Fig. 5). On substituting the polynomial approximation for the optimum thermal conductivity

$$k = a_0 + a_1 T + a_2 T^2 \tag{33}$$

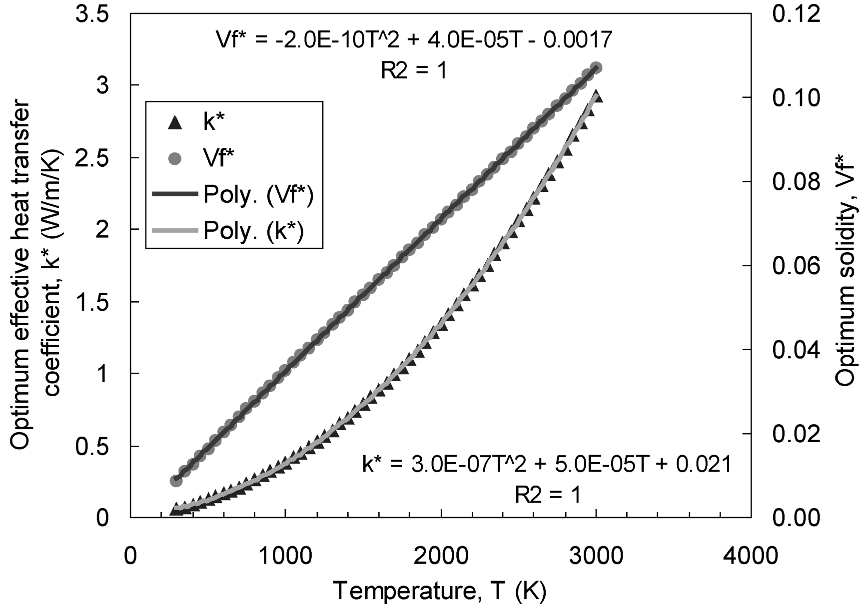


**Fig. 5 Optimum effective heat transfer coefficient (conductivity) and volume fraction (solidity) of titanium open-cell foam as function of temperature and quadratic polynomials fitted to data.**

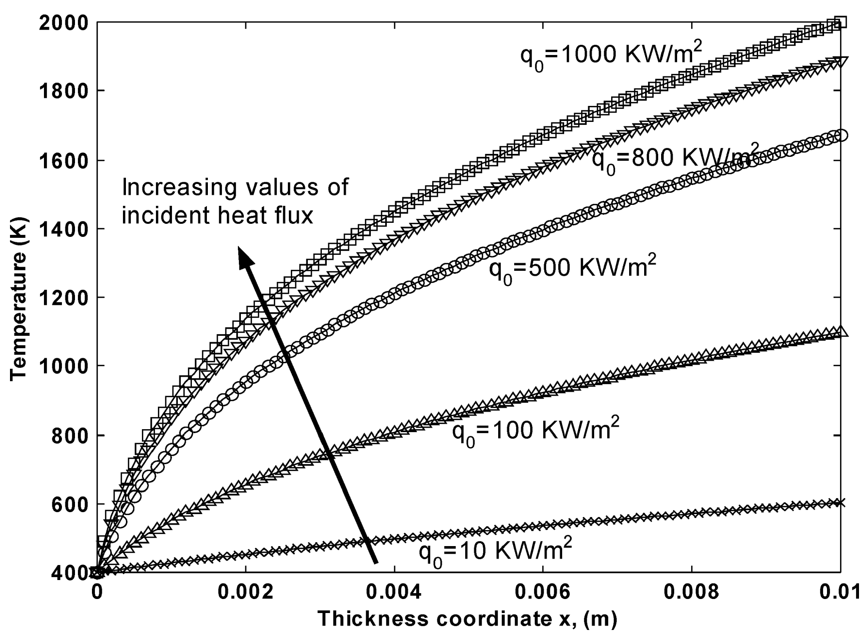


**Fig. 6 Temperature distribution through the thickness of the insulation for different values of incident heat flux that satisfy optimality conditions for maximum outside temperature.**

into the heat transfer equation and integrating in through the thickness, we obtain the following algebraic equation which is solved to obtain $T_h$:

$$\left[a_0 T_h + (a_1/2)T_h^2 + (a_2/3)T_h^3\right] + h\sigma T_h^4 = Q_0 h + \left[a_0 T_0 + (a_1/2)T_0^2 + (a_2/3)T_0^3\right] \tag{34}$$

Once $T_h$ is known, the heat transfer equation is integrated to obtain the temperature profile to obtain it the optimum density profile, $V_f^*(t)$, through the insulation.

The temperature distribution in the insulation of fixed thickness $h = 0.01$ m, inside temperature $T_0 = 400K$, and different values of incident heat flux, from $0.01 \times 10^6$ to $1 \times 10^6$ W/m$^2$ are calculated using the expression derived earlier and plotted in Fig. 6. Figure 6 is as expected, that is, when the thickness $h$ and inside wall temperature $T_0$ are kept constant and heat flux at surface is increased, the outside temperature increases. At higher temperatures, the temperature distribution is nonlinear because the radiation component becomes significant. The distribution of cell size or solidity corresponding to the temperature distribution (Fig. 6) can be calculated using the polynomial expressions for solidity shown in Fig. 5. These optimum through the thickness distributions of volume fraction are shown in Fig. 7. The optimum graded density insulation is compared with uniform density insulation for different values of the incident heat flux. Figure 8 shows the ratio of the maximum temperature attained at the outside wall by the optimized graded density and uniform density insulation. The graded density insulation results in 0.3–0.6% higher temperatures. These small differences in temperature result in large differences in the heat radiated at the outside wall because radiation is proportional to the fourth power of temperature. An accurate measure of performance of thermal insulation is the fraction of the heat that is transmitted to the

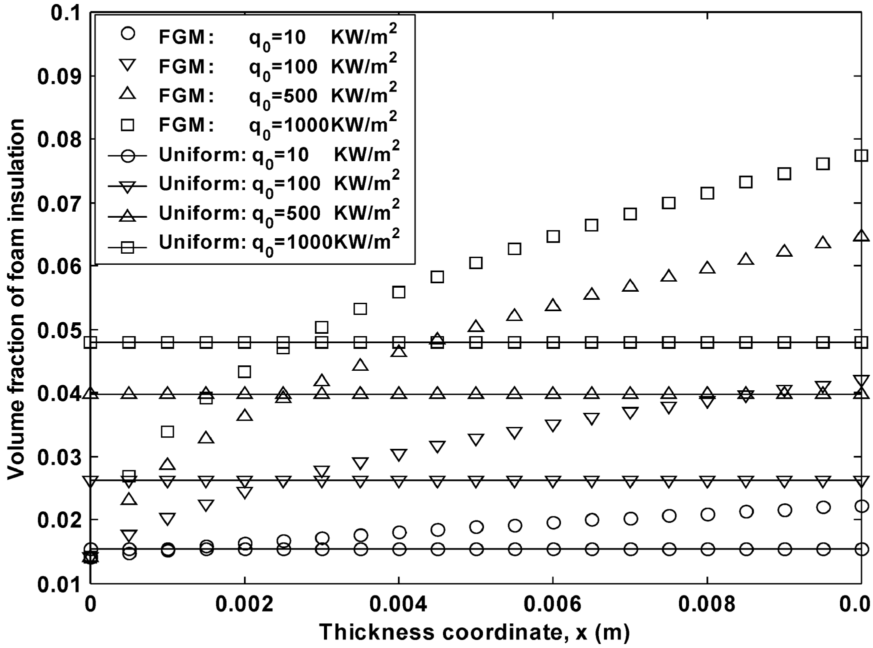


**Fig. 7 Comparison of volume fraction (density) distributions obtained for the optimum graded and optimum uniform density insulation for different values of incident heat flux.**

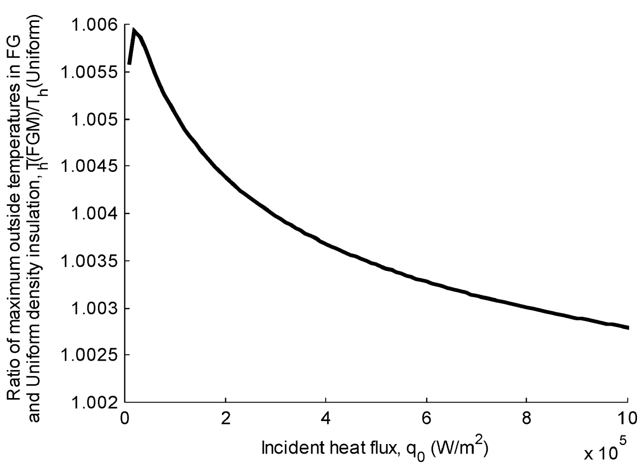


**Fig. 8 Comparison of maximum temperature attained using optimum constant density and optimum functionally graded density foams.**

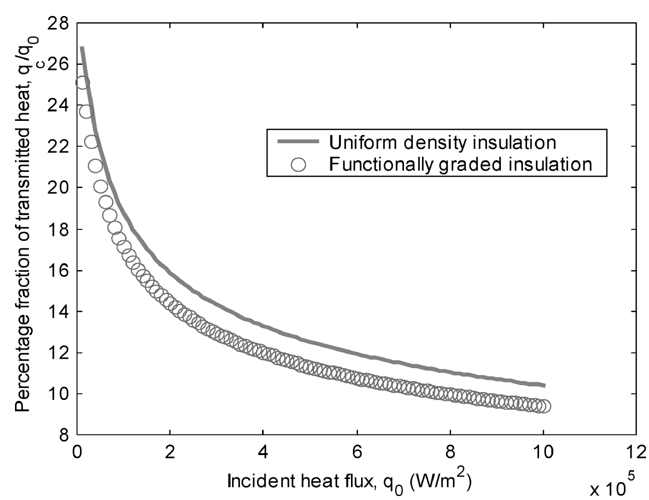


**Fig. 9 Comparison of heat transmitted through open-cell foam insulation of optimum uniform and optimum functionally graded density.**

inside. Ideally, we want to reduce this to be as low as possible. Figure 9 presents the percentage of heat transmitted through the foam insulation for different values of the incident heat flux on the outside surface. The fraction of heat transmitted through the optimum graded density foam insulation is compared to that in the optimized uniform density foam insulation. The uniform density design was obtained by numerical maximization of outside wall temperature by varying the (single variable) foam density. Comparisons of the transmitted heat for the two designs indicate that the functional grading in insulation density reduces the heat transfer by 8–10%.

Figure 10 shows the ratio of the thicknesses of functionally graded density insulation and uniform density insulation. At low heat flux condition, the conduction mode dominates the heat transfer. The FGM foam can reduce the density of the foam by increasing the cell size in the low-temperature region of the foam. The uniform insulation increases the foam cell size throughout the thickness, thereby incurring a larger increase in thickness. At higher heat flux conditions where radiation becomes dominant, the FGM reduces heat transfer by reducing cell size at the outside wall (hotter region). The uniform density foam cannot reduce the cell size as effectively as the FGM foam because the small cell size significantly increases the conduction heat transfer at the inside wall. The cell size chosen for the uniform density foam is a compromise between cell sizes needed for minimizing conduction and radiation mode heat transfer. The optimum density reduces the thickness of the insulation between 9 and 12.5% depending on the value of the heat flux. The thickness of the graded insulation combined with the expression for the optimum volume fraction (Fig. 5) was used to calculate the mass of the equivalent graded thermal insulation. The ratio of the mass of the graded density and uniform density of identical performance is plotted in Fig. 11 for open-cell foam. The reduced thickness of the thermal insulation comes with a small increase in weight (4.5–9.0%). Similar calculations were repeated for closed-cell foams in the Appendix. These showed that for closed-cell foams the gains in performance are miniscule. This is because in closed-cell foams the conductivity of the foam solid material is a significant portion of the effective conductivity.

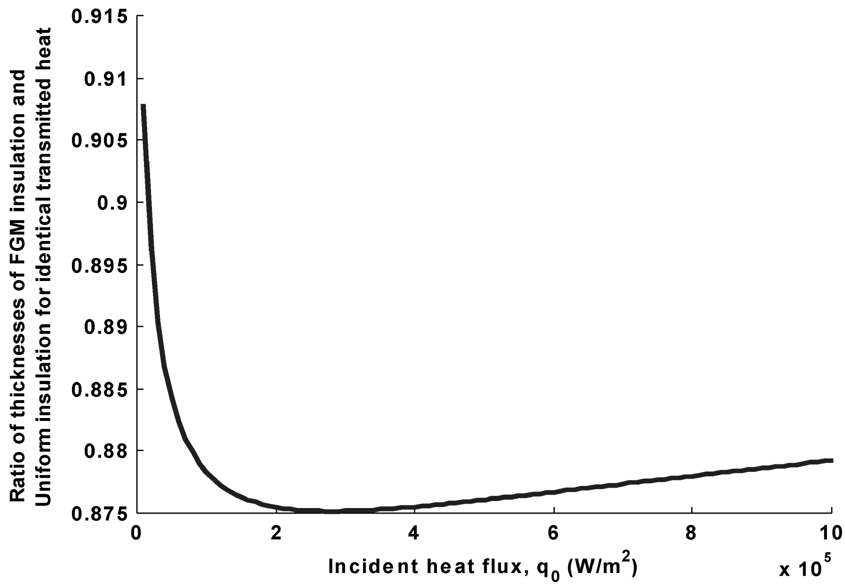


**Fig. 10 Ratio of optimum functionally graded density insulation thickness compared to the optimum uniform density insulation thickness.**

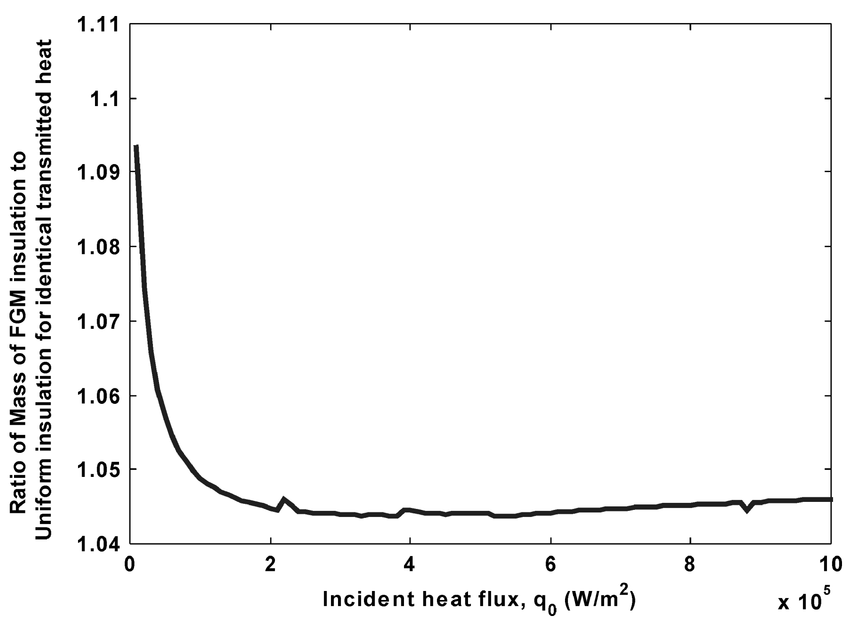


**Fig. 11 Mass of optimum functionally graded insulation compared to uniform insulation that exhibits identical performance (same surface temperatures).**

## Summary

Optimum density distribution of a thermal insulation for steady-state heat conduction is studied for open- and closed-cell foams. The effective thermal conductivity of the foam is derived in terms of the strut diameter, the cell size, and the temperature. Maximizing the temperature at the outside wall of the insulation, which maximizes the heat radiated, minimizes the heat transmitted through the insulation. An optimality condition is derived, and the optimization problem is reduced to that of an ordinary, but a nonlinear differential equation, which is solved numerically. The same optimality condition is also found to hold for minimizing the thickness of the insulation. In open-cell foams, the heat transfer mode is predominantly radiation at high temperatures and conduction at low temperatures. For any given temperature, there is an optimum density that minimizes the heat conductivity through the foam. The optimal density distributions reduce the heat transmitted through the closed-cell foam insulations by up to 10% compared to the optimum uniform density insulation. The minimum thickness functionally graded insulation is up to 12% thinner compared to the uniform density; however, it is slightly heavier. In the closed-cell insulation, heat transfer is predominantly by conduction and, hence, does not show significant benefit in tailoring the density distribution.

## Appendix: Equations and Results for Closed-Cell Foam Insulation

In this Appendix, we develop the heat transfer equations for closed-cell foam. The optimality condition derived in the paper is applied to closed-cell foams to calculate performance measures such as heat transmitted through the foam and the minimum foam thickness for a given transmitted heat.

### Effective Thermal Conductivity

The model assumes the cells are hollow rectangular prisms having dimensions ($a \times a \times b$) and wall thickness of $t$. The expression for the solidity for the rectangular closed-cell foam shown in

Fig. A1 is

$$V_f = \frac{\text{volume in walls}}{\text{total volume}} = \frac{(2a^2 + 4ab)(t/2)}{a^2 b} = \frac{t}{a}\left(\frac{a}{b} + 2\right) \quad \text{(A1)}$$

This simplifies to the following form for a cubic cell of size $a$:

$$V_f = 3(t/a) \quad \text{(A2)}$$

**Gas Conduction**

The gas conduction equation presented for open-cell foams is valid for closed-cell foams also. However, the characteristic dimension $l_c$ is the size of the cell,

$$l_c = a \quad \text{(A3)}$$

**Solid Conduction**

The heat conduction path in closed-cell foams is through the cell walls. For the idealized model used here, we assume there is no concentration of mass at the edges (struts) and all mass is uniformly distributed in the cell walls. For this assumption, the simple conduction model developed by Glicksman [Eq. (5.38) in Ref. 3] when expressed using the present notation is

$$k_s = \tfrac{2}{3} k_m^* V_f \sqrt[4]{(a/b)} \quad \text{(A4)}$$

where $k_m^*$ is the temperature dependent conductivity of the solid material (metal) in the foam.

**Radiation**

The expression for radiation heat transfer in idealized closed-cell foam with no struts as obtained from Glicksman [Eq. (5.60) in Ref. 3] is

$$k_r = (16/3)\left(\sigma T^3 b \big/ 4.1\sqrt{V_f}\right) \quad \text{(A5)}$$

where $k_r$ is in watts per meter per degree Kelvin and $\sigma$ is the Stefan–Boltzmann constant.

The effective transfer coefficient is calculated as before by adding the three terms. The dependence of the heat transfer coefficient on temperature and volume fraction is shown in Fig. A2. The dependence of the effective heat transfer coefficient on temperature and density (volume fraction) for closed-cell foam is qualitatively similar to open-cell foam, despite the differences in the influence of cell size on their heat transfer modes. At each temperature, there is an optimum value of volume fraction that minimizes the heat transfer equation. The optimum volume fraction and the corresponding value of the heat transfer coefficient were calculated numerically using the heat transfer equation presented in the paper with modifications mentioned earlier for closed-cell foams. The numerical obtained values of optimum volume fraction and heat transfer coefficient are fitted with polynomial functions that describe their dependence on temperature (Fig. A3).

The temperature distribution through the thickness of the insulation is obtained using the polynomial expression for the optimum effective heat transfer coefficient in the heat conduction equations as presented before by integrating the heat transfer equation. The outside wall temperature is then used to compute the amount of heat radiated at the outside surface on which the aerodynamic heating occurs. The heat transmitted as a fraction of the incident heat flux is calculated for different values of the incident heat flux (Fig. A4). For identical values of incident heat flux, the heat transmitted thought the best uniform density insulation is also calculated and compared to the nonuniform (functionally graded) density foam insulation. For closed-cell foam, using varying density provides only minuscule improvement to the thermal insulation performance. Figure A4 also shows the heat transmitted through the open-cell foam insulation is

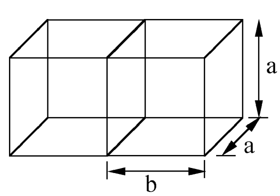


**Fig. A1 Unit cell of idealized closed-cell foam; each wall uniform thickness *t*, direction with cell dimension *b* indicates heat transfer direction.**

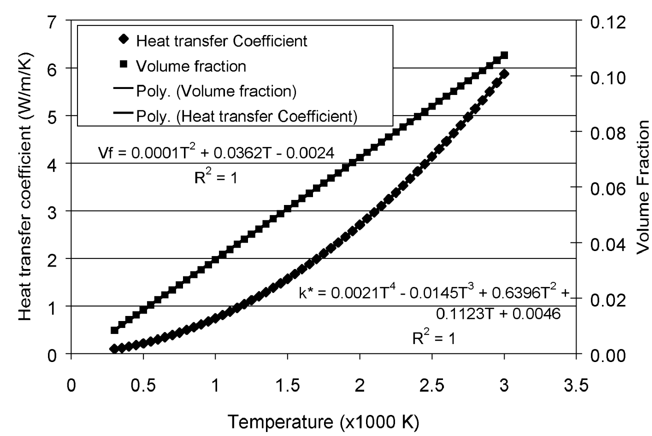


**Fig. A3 Optimum effective heat transfer coefficient (conductivity) and volume fraction (solidity) of titanium open-cell foam as function of temperature and quadratic polynomials fitted to data.**

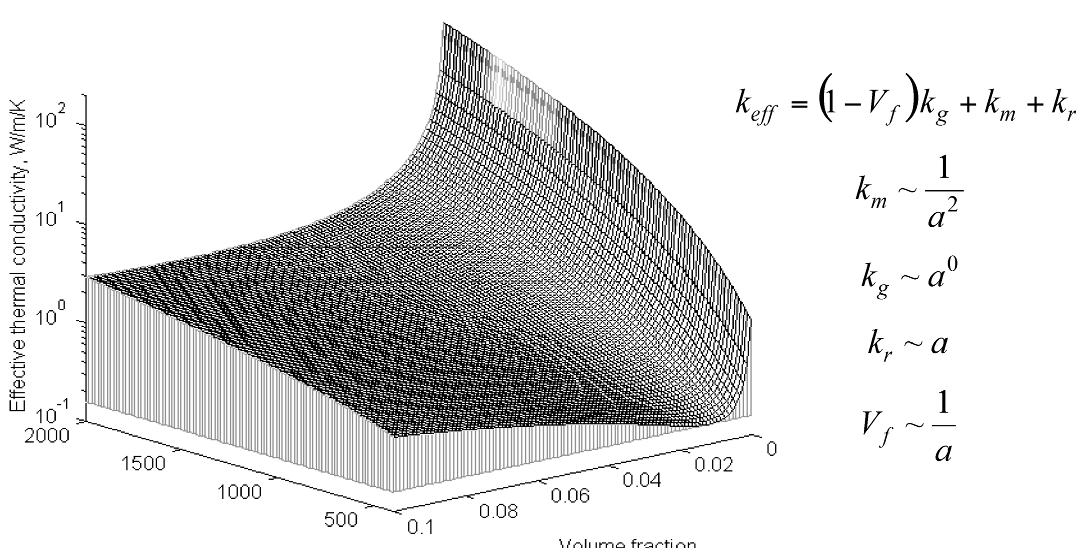


**Fig. A2 Effective thermal conductivity for range of temperatures *T* and volume fraction $V_f$ and dependence of the components of the effective conductivity on foam pore size obtained for titanium closed-cell foam with fixed wall thickness of 0.05 mm.**

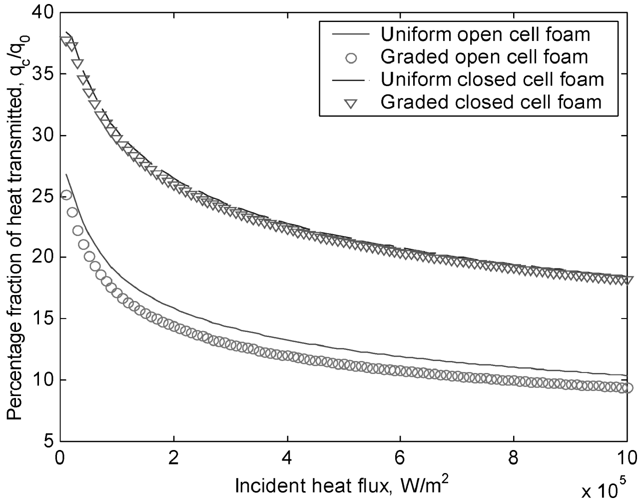


**Fig. A4 Comparison of heat transmitted through open-cell and closed-cell foams with uniform and functionally graded density through the thickness in the insulation.**

significantly lower than the closed-cell foam insulation. In the case of closed-cell foam, conduction is the dominant heat transfer mode and, therefore, does not benefit from varying the density through the thickness.

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