Fracture Toughness of Cellular Solids using Finite Element Based Micromechanics

J. WANG AND B. V. SANKAR

ABSTRACT

Micromechanical methods to predict the fracture behavior and estimate the fracture toughness of cellular solids and foams are presented in this paper. The microstructure of the foam is modeled using finite elements. The struts of foam are modeled using either structural elements such as beams or three-dimensional solid elements. A portion of the foam containing an edge crack is modeled. Displacement boundary conditions are applied such that they correspond to a given value of stress intensity factor in the homogenized solid. The crack propagation is simulated by breaking the crack tip strut when the maximum stress in that strut exceeds the strength of the strut material. The results show that the displacements and stresses in the foam near the crack tip are very similar to that in an equivalent homogeneous material, and continuum fracture mechanics concepts can be applied to predict the fracture of a cellular medium. Numerical results are presented for Mode I fracture toughness tetrakaidecahedral foams.

INTRODUCTION

Cellular materials are made up of a network of beam or plate structures leaving an open space or cell in between. Cellular materials, e.g., carbon and polymeric foams, offer several advantages such as thermal resistance, durability, low density, impact damage tolerance and cost effectiveness. They have great potential as core materials in sandwich construction, which has application in heat exchangers and thermal protection systems in military and commercial aerospace structures.

An excellent treatise on the structure and properties of cellular solids has been written by Gibson and Ashby [1]. While analytical methods for predicting thermal and thermo-mechanical properties of cellular media are well documented, research on fracture behavior of various foams is still in its infancy. Simplified approximate

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Junqiang Wang, Graduate Student, currently with Caterpillar Tractor Co., Decatur, IL, USA
Bhavani V. Sankar, Professor, Department of Mechanical & Aerospace Engineering, University of Florida, Gainesville, FL

formulas for Mode I fracture toughness of cellular solids in terms of their relative density and tensile strength of the strut or ligament material are available. However, these are limited to cracks parallel to the principal material directions. Moreover, fracture behavior under mixed mode has not studied. In order to estimate the fracture toughness, the stresses in the crack-tip strut (first unbroken cell edge) is calculated in terms of the global stress intensity factor. Then the maximum stress in the strut due to the bending moment is equated to the tensile strength of the strut material. The stress intensity factor that would produce such a bending moment is taken as the fracture toughness of the foam.

In this study, the cellular solid on a macro-scale is considered as a homogeneous orthotropic material. A crack parallel to one of the principal material directions is assumed to exist in the solid and a small region surrounding the crack tip is modeled using Euler-Bernoulli beam finite elements. The commercial finite element software ABAQUS® was used for this purpose. The strut material is assumed to be isotropic, linearly elastic and brittle, and its elastic constants and tensile strength are assumed known. The crack is modeled by breaking several struts along the line of the intended crack. Results rare presented for foams with rectangular lattice and tetrakaidecahedral microstructures. Details of the micromechanical models for various types of foams can be found in [2 through 5]. A brief account of the procedures and results are presented here.

FINITE ELEMENT MODEL

In this section, we describe a finite element based micromechanics model for estimating the fracture toughness of the cellular solid. Mode I, Mode II and mixed mode fracture conditions are considered. Both cracks parallel to the principal material axes and inclined cracks are considered. To determine the fracture toughness, a small region around the crack tip is modeled using beam elements, and a constant mode mixity K_I/K_{II} is considered. The boundary of the cellular solid is subjected to displacement boundary conditions u_1 and u_2 corresponding to an arbitrary value of K_I (or K_{II}). The calculation of boundary displacements for a given stress intensity factor is described in the next section.

Boundary displacements near the crack tip

The displacement components in the vicinity of a crack tip in a homogenous orthotropic material are as follows [3]:

The displacement filed near the crack tip for Mode I:

$$u_1 = K_I\sqrt{\frac{2r}{\pi}}\,\mathrm{Re}\left\{\frac{1}{s_1-s_2}\left[s_1p_2\left(\cos\theta+s_2\sin\theta\right)^{1/2}-s_2p_1\left(\cos\theta+s_1\sin\theta\right)^{1/2}\right]\right\}$$

$$u_2 = K_I\sqrt{\frac{2r}{\pi}}\,\mathrm{Re}\left\{\frac{1}{s_1-s_2}\left[s_1q_2\left(\cos\theta+s_2\sin\theta\right)^{1/2}-s_2q_1\left(\cos\theta+s_1\sin\theta\right)^{1/2}\right]\right\}$$

The displacement filed near the crack tip for Mode II:

$$u_1 = K_{II}\sqrt{\frac{2r}{\pi}}\,\mathrm{Re}\left\{\frac{1}{s_1 - s_2}\left[p_2\left(\cos\theta + s_2\sin\theta\right)^{1/2} - p_1\left(\cos\theta + s_1\sin\theta\right)^{1/2}\right]\right\}$$
$$u_2 = K_{II}\sqrt{\frac{2r}{\pi}}\,\mathrm{Re}\left\{\frac{1}{s_1 - s_2}\left[q_2\left(\cos\theta + s_2\sin\theta\right)^{1/2} - q_1\left(\cos\theta + s_1\sin\theta\right)^{1/2}\right]\right\}$$

In deriving the above expressions, the crack is assumed to be parallel to the 1-axis, and r-θ is the polar coordinate system situated at the crack tip. The complex parameters p, q, and s depend on the elastic constants of the homogeneous orthotropic material as described in [3]

Finite Element Model

The commercial program ABAQUS™ is used to perform the finite element analysis. A portion of the foam surrounding the crack tip is modeled using beam finite elements. The displacement boundary conditions for the FE model are determined from the expression for boundary displacements described in the previous section. The crack in the FE model is created by removing the beam elements along the line of the crack behind the crack tip.

The maximum tensile stress in the struts is calculated from the finite element method. The FE analysis outputs axial force, bending moment and shear force at each node of the beam elements, and the maximum tensile stress was calculated using a separate program. Usually the maximum stress occurs at the crack tip strut. It should be noted that the stresses in the struts vary linearly with respect to the applied stress intensity factor K_{I} (or K_{II}). Since we know the strength of the strut material, the value of K_{I} (or K_{II}) that will cause rupture of the strut can be estimated. And then, it is taken as the fracture toughness of the cellular solid. It should be mentioned such scaling is possible because we assume linear elastic behavior of the strut material and hence that of the foam. If the strut material undergoes inelastic behavior, then an iterative method will have to be used to determine the stress intensity factor that will cause the failure of the crack-tip strut.

RESULTS

Tetrakaidechahedral foams

The tetrakaidecahedral unit cell that we propose to study is a 14-sided polyhedron with six square and eight hexagonal faces. It is more precisely called truncated octahedron, since it is created by truncating the corners of an octahedron from a different viewpoint, it can be generated by truncating the corners of a cube. All the edges of the cell are of equal length L and cross sectional area A.

The tetrakaidecahedral foam has a BCC lattice structure. The axes of the BCC lattice are parallel to the axes of the cube. Due to the symmetry of the structure, the Young's moduli of the foam in the lattice vector directions are equal:

$$E^*_{001} = E^*_{010} = E^*_{100}$$

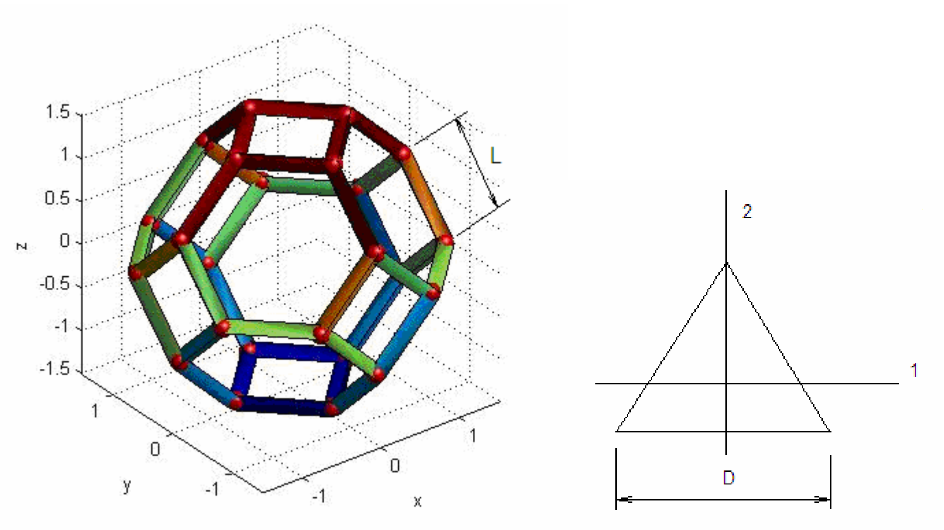

Figure 1: A tetrakaidecahedral unit cell and the cross section of a strut

Each strut of the cell is treated as a beam element. In our study, the cross section of the struts is assumed to be an equilateral triangle with side length *D* (Figure 1). The material properties of the solid material are: Young's modulus=23 GPa, Poisson's Ratio=0.33 and tensile strength = 690 MPa. A model used to determine the leastic properties is shown in Figure 2. Two convergence studies were conducted: Case 1 in which the cell number is increased gradually in both *x* and *y* directions; Case 2 in which the cell number in the *x* direction is increased and that in the *y* direction is kept constant. The results are shown in Figure 3**.**

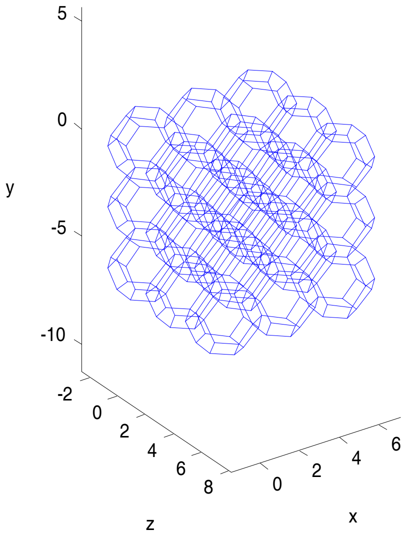

Figure 2: Model of the tetrakiadecahedral foam with 27 cells (3×3×3)

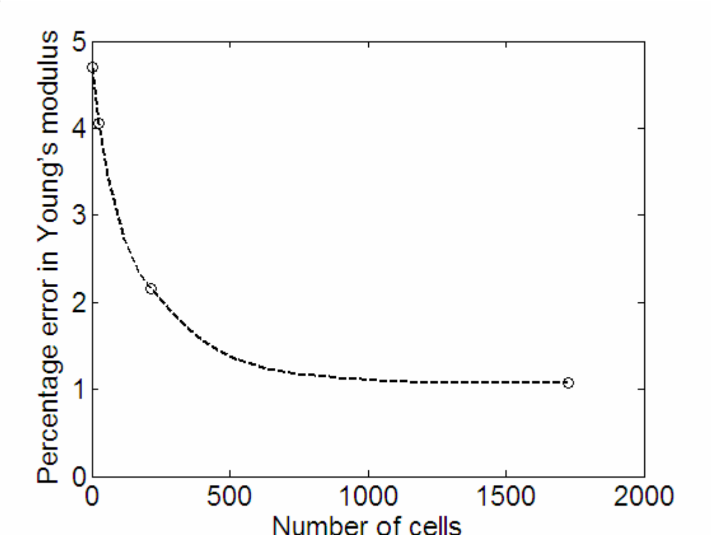

Figure 3: Convergence study for Young's modulus.

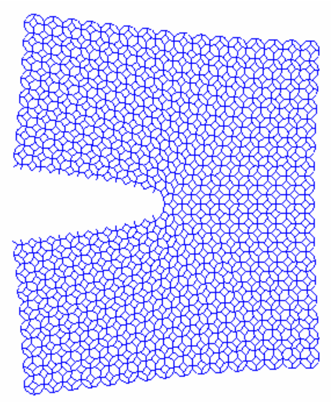

Figure 4: Deformation of a micromechanical model

The fracture toughness was calculated for various number of unit cells until convergence was achieved. The deformation of the micromeodel is shown in Figure 4. The results are presented in Tables 1 and 2, and presented in Figure 5. One can note that the K_{Ic} values of the tetrakaidecahedral foam converge to a constant value with increasing number of cells in the model.

Table 1: Convergence of fracture toughness; the number of cells in the *x* and *y* directions are varied

Number of Cells	10×5 50	16×8 128	20×10 200	25×12 300	32×16 512
K_{Ic}	0.405	0.399	0.397	0.395	0.393

Table 2: Convergence of fracture toughness; the number of cells in the *y* direction is kept at 10, and that in the *x* direction is varied.

Number of Cells	10×10 100	20×10 200	30×10 300	40×10 400	50×10 500
K_{Ic}	0.405	0.397	0.394	0.393	0.392

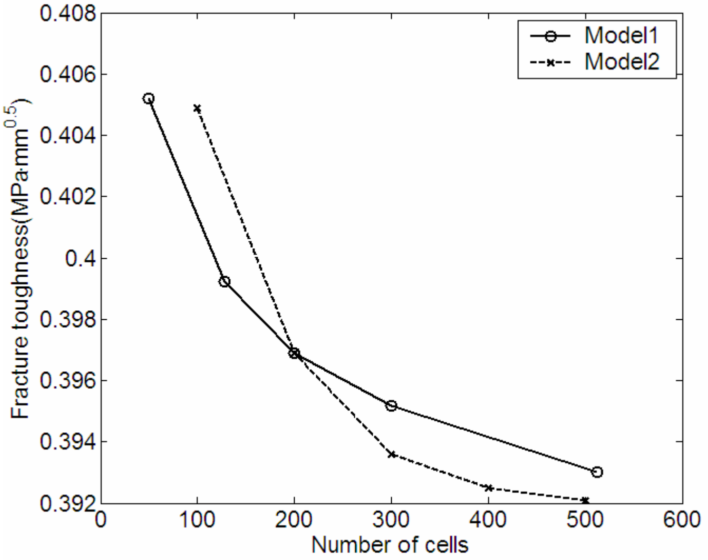

Figure 5: Convergence of mode I fracture toughness with the number of cells

CONCLUSIONS

A finite element based micromechanics method has been developed to determine the fracture toughness of cellular materials. A portion of the cellular medium surrounding the crack tip is modeled using beam finite elements. Displacement boundary conditions are applied such that they correspond to a given value of stress intensity factor in a homogeneous solid that has the same elastic constants as the cellular medium. The stresses developed in the beam elements (struts) are used to determine if the strut will break or not. From the results the fracture toughness of the cellular medium is estimated. It is shown that the value obtained is independent of the crack length or number of cells used in the model indicating that the calculated value of the fracture toughness is truly a material property of the foam.

ACKNOWLEDGEMENT

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