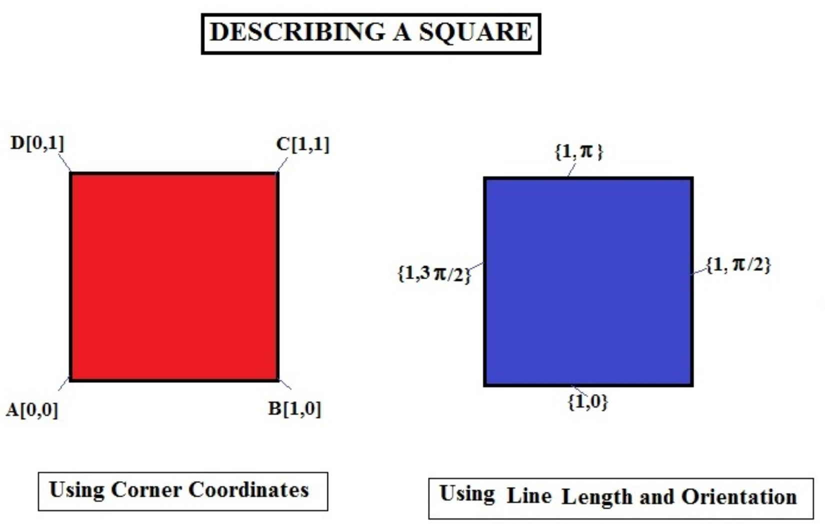
CONSTRUCTION OF 2D CURVES USING STRAIGHT LINE INCREMENTS

It is well known that any 2D curve can be thought of as the concatenation of a series of incremental straight lines each oriented in a specified direction. Thus the square can be defined as the connection of four unit length lines hooked together at their ends at ninety degrees to each other as shown-



You will notice that the length–angle definition for the blue square works just as well as specifying the coordinates of the square vertexes as done for the red square. The advantage of the blue square definition is that it can bedescribed by a genetic algorithm in which any 2D curve is definedby the elements –

$$\{L,\theta\}=\{[L_1,\theta_1], [L_2,\theta_2],\ldots\}$$

with [L_{n+1},θ_{n+1}] related to[L_n,θ_n]. The genetic algorithm for a square of unit length sides will be just-

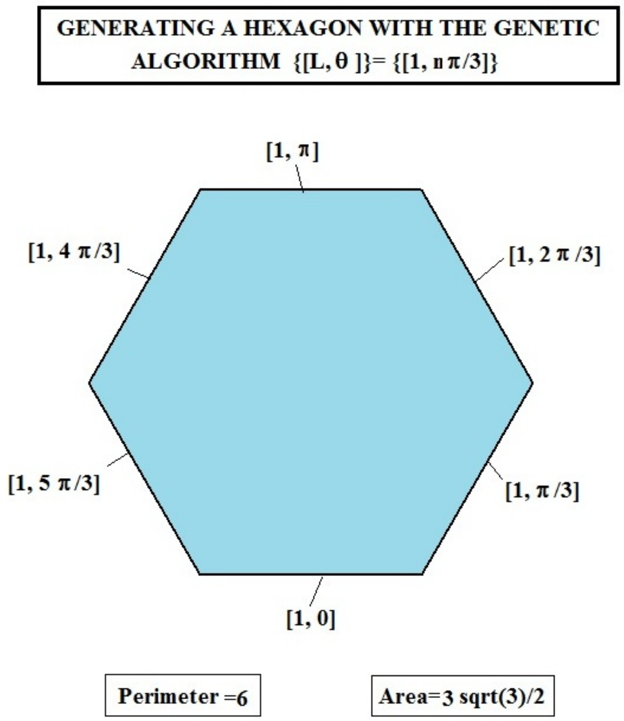
$$\{[1, n\pi/2]\} \quad n=0, 1, 2 \text{ and } 3$$

If we want to increase the side-length to L, one simply replaces 1 by L. To rotate the figure by φ radians one needs to just replace nπ/2 by nπ/2+φ. Letting L become very small, allows one to approximate any continuous curve by the concatenation of such incremental line lengths. This is essentially what your computer does when plotting a continuous curve.

The genetic algorithm approach works especially well for all regular polygons. The algorithm for an n sided regular polygon, of side-length one each, is -

$$\{ L,\theta\}=\{[1, 2\pi/n]\}$$

Thus taking $\theta=\pi/3$ produces the following hexagon-



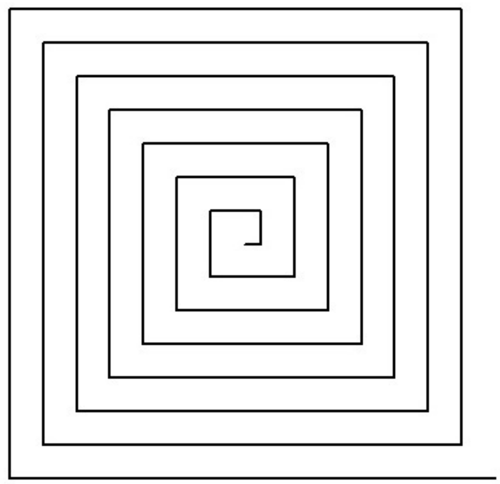
The perimeter of this hexagon is just n= 6.

It is not always necessary to keep L fixed. By letting it vary one can generate spirals. Take, for example, the case –

$$\{[L,\theta]\}=\{[n+1, n\pi/2]\} , n=0, 1,2,\ldots$$

Here the first length increment is $L_0=1$ and its orientation is horizontal pointing east. This is followed by the next line increment $L_1=2$ going north. Concatenating the first 29 line elements produces the following figure-

RECTANGULAR SPIRAL GENERATED BY {[n+1,n π /2]} , n=0, 1, 2,.. , 28.



The total length shown equals-

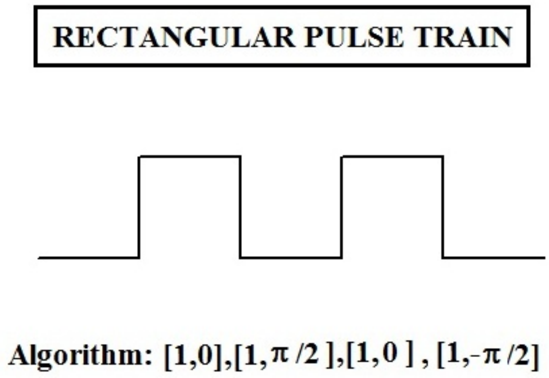
$$L = \sum_{n=0}^{28} (n+1) = 435$$

The figure is reminiscent of the crystal growth pattern sometimes observed in the vicinity of a dislocation.

The above spiral when taken to infinity is unbounded. There are many additional 2D curves which have this property. Consider for example the genetic algorithm-

$$\{[L,\theta]\}=\{[1,0],[1,\pi/2],[1,0],[1,-\pi/2], [1,0],[1,\pi/2],[1,0],[1,-\pi/2],\ldots\}$$

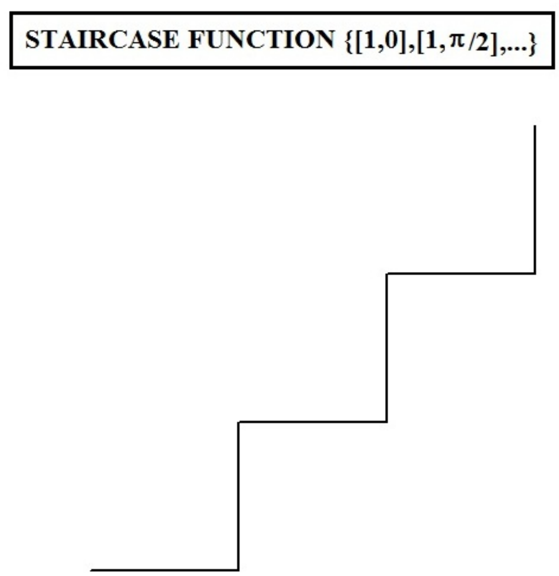
This shows the basic four element pair [1,0],[1,π/2],[1,0],[1,-π/2] repeating indefinitely . The resultant figure is the rectangular pulse pattern of infinite extent. Part of it is shown in the following-



Just a small change in the code to-

$$[1,0],[1,\pi/2],[1,0],[1,\pi/2],\ldots$$

produces the two pair staircase function shown-



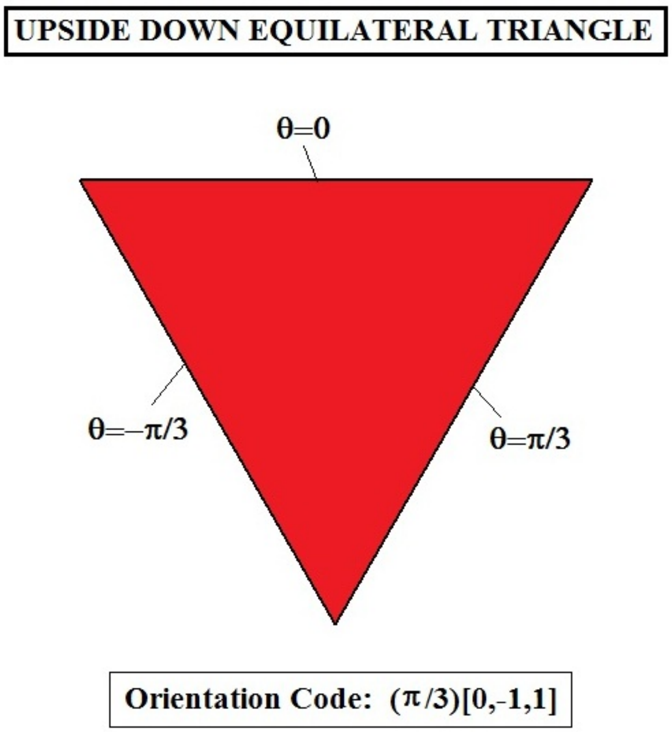
One can dispense with the length portion of an increment length-orientation pair when L_n is kept constant throughout. So one could define any equilateral triangle of unit side-lengths as-

$$(2\pi/3+\varphi)[\ 0,1,-1]$$

, where φ represents is the rotation angle which the triangle is rotated by from its original horizontal base. So an upside down equilateral triangle has the orientation of its sides given by-

$$(\pi/3)[0,-1,1]$$

A graph of the resultant triangle follows-

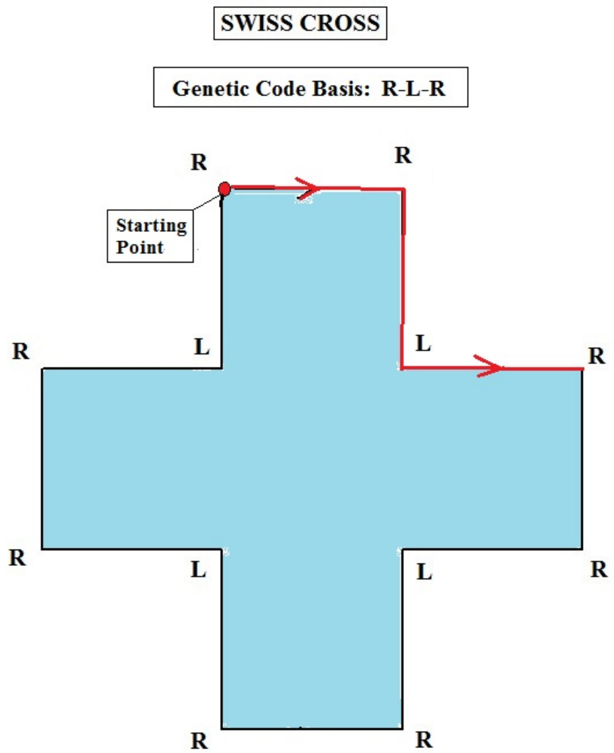


In deriving these results use was made of the cyclic nature of the angle measure which says $\theta=\theta+2\pi n$.

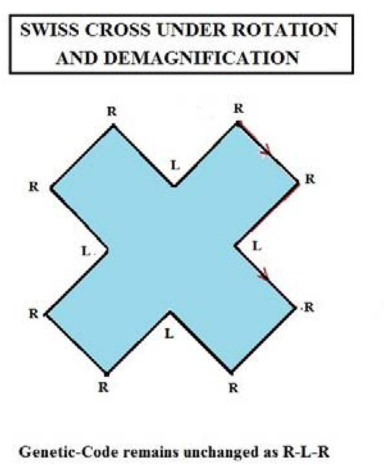
Finally we point out that all of the above 2D curves have relied an a fixed coordinate system with specified origin. A genetic code really does not require such information and it should be sufficient to define any straight line segment with just the length of the segment and the angle with which the next segment of specified length is attached to it. If all segments are of equal length and the connection to an element's neighbor is either by a right angle or zero angle, then a designation involving left(L), straight(L), and right(R) should be sufficient to describe the entire figure. Consider the code-

{R-L-R-R-L-R-R-L-R- …..}

This code repeats the triplet R-L-R indefinitely. Plotting it one finds the Swiss Cross as shown-

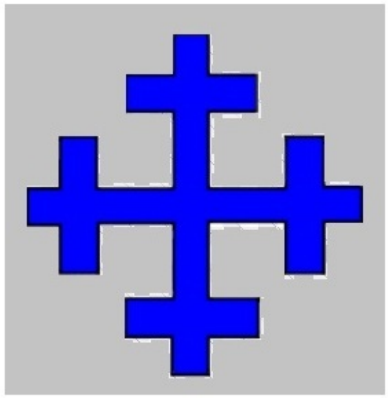


This code will reproduce this cross for any desired rotation angle and any desired line-segment length. That is , the code is independent of the coordinate system being used. A $\varphi=\pi/4$ rotation and a halving of the line segments produces the following-



Very intricate 2D straight-line segment curves can be produced by codes involving just the L,S,R letters. Here is an example of a crosslet-

BLUE CROSSLET



Code repeated four times: [S-L-R-R-L-R-R-L-R-R-L-S]

The genetic code involving just the tree letters L,S,R is indicated at the bottom of the figure. This cross emerged during the crusades in the twelve hundreds and later morphed into the well known Teutonic Cross found on modern day German military aircraft.

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