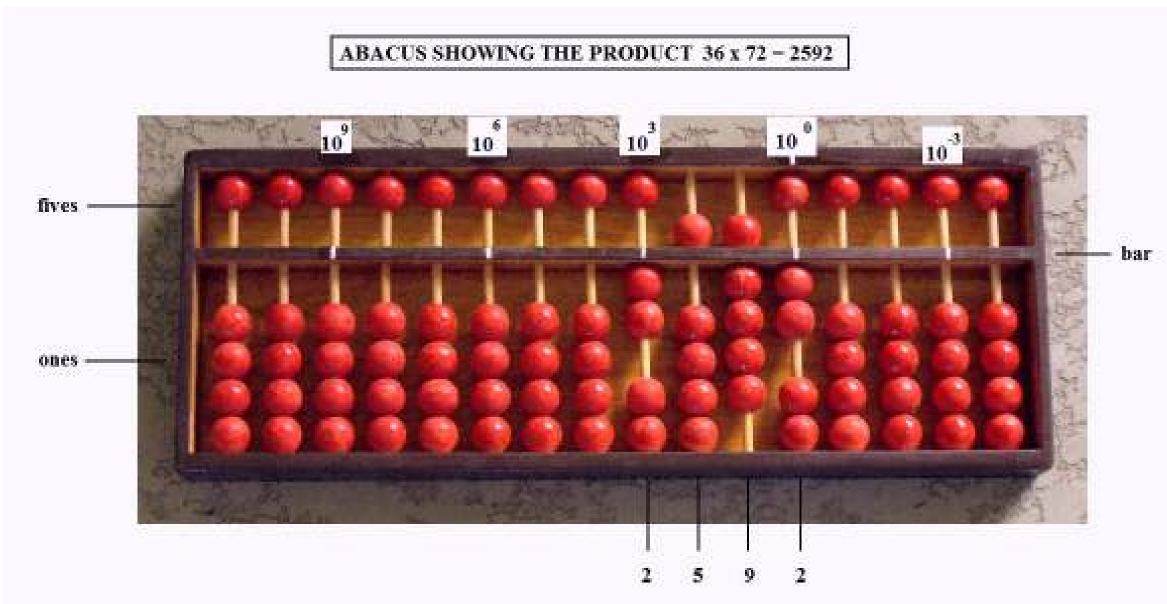
HOW CAN THE ABACUS BE USED TO INCREASE ONE'S ABILITY TO QUICKLY ADD AND MULTIPLY?

The Abacus is one of the oldest known mathematical calculators going back to the ancient Chinese. It consists essentially of a series of parallel sticks along which beads are free to slide. It is a digital device so, unlike the slide rule, it produces precise answers provided the size of the numbers being manipulated stay within the range for which the Abacus is designed. I show you here a typical Abacus. It is one which I recently constructed in my worshop using ¾ inch wood beads obtained at a local crafts store made to slide along 3/16 inch diameter dowels.-



In this particular model I have 16 columns of beads with the column designation going from 10^{-4} on the right through 10^{11} on the left. All beads which are moved toward the center bar are counted as a one when coming up from the bottom section and five when moved down from the top section. The number showing on the above picture is 2592=2000+500+90+2. This number happens to be the product of 36 and 72. We will show this fact latter. What should already be apparent is that the Abacus requires the operator to understand how many thousands, hundreds, tens, etc a number contains. This is not always clear to students operating a hand calculator and thus prevents many from making quick mental calculations using an approach similar to that used with the Abacus.

Let us begin with the basic mathematical process of addition. To demonstrate consider summing the three numbers 2345, 5783, and 3948. Decomposing things as one does with an Abacus, we have (2+5+3) in the thousands group, (3+7+9) in the hundreds group, (4+8+4) in the ten group, and (5+3+8) in the ones group. That is-

$$16+160+1900+10000=12{,}076$$

This is a very easy procedure for an operator to set up on an Abacus and to carry out the calculation. Some of you may have noticed that there still are a few Chinese restaurant owners who figure out your dinner bill this way and do so at hand calculator speed. Subtraction of two numbers is treated in a similar manner. Take –

$$3425-2798=7+20+600=627$$

Lets next turn to multiplication. Take two 2 digit generic numbers and expand them as shown-

$$AB\,x\,CD=AC\,x10^2+(AD+BC)\,x10^1+BD$$

The Abacus operator will set up AC in the hundreds column, AD+BC in the tens column, and BD in the ones column. If any of the products exceed 9 then this excess is added to the next column to the left. Again a very easy operation. It also is a recipe for quick mental calculations. Using the above breakup, one has at once that-

$$73\,x\,47=2800+(49+12)10+21=3431$$

With a little practice this type of calculation can be done in one's head and done so at speeds in excess of those required by the standard multiplication approach learned in school. The special case of squaring a number follows from-

$$(AB)^2=100A^2+20AB+B^2$$

Thus 99x99=8100+1620+81=9801. Of course one could get the same result even simpler by just subtracting 99 from 9900. One also has 33x33=9801/9=121x9=1089 by noting that 11x11=121 and 3x3=9.

When dealing with multiplication of three digit numbers, one finds the generic result-

$$ABC\,x\,DEF = ADx10^4 + (AE + BD)x10^3 + (DC + EB + AF)x10^2$$
$$+ (EC + BF)x10^1 + CF$$

Thus 236x892=160,000+42,000+7900+600+12=210,512. Squaring a three digit number is even easier since-

$$(ABC)^2 = A^2x10^4 + 2ABx10^3 + (B^2 + 2AC)x10^2 + 2BCx10^1 + C^2$$

So if the number you want to square is N=125, you can say that-

$$N^2 = 10{,}000 + 4000 + 1400 + 200 + 25 = 15{,}625$$

There are, of course, other and sometimes simpler means to calculate squares. Here are a few more for squaring N=125-

$$(125)^2 = (120 + 5)^2 = 14400 + 1200 + 25 = 15{,}625$$
$$(125)^2 = (625/5)^2 = 25^3 = 62500/4 = 15{,}625$$
$$(125)^2 = (10^2 + 5^2)^2 = 5^6 = 15{,}625$$

What this last result shows is that one must remain flexible with the approach to multiplication one takes. Most often there will be a way which is much faster than using the 'standard'method. When working out products in ones head, on paper, or via the Abacus, it always pays to make use of knowledge such as when a number is a power of another number or a number lies near a simpler number to multiply.

Consider finding the square of the product 16*128. Here we have-

$$(16 \cdot 128)^2 = (2^{11})^2 = 2^{22} = 64 \cdot 256 \cdot 256 = 4{,}194{,}304$$

Also-

$$78 \cdot 168 = 80 \cdot 168 - 336 = 13{,}440 - 336 = 13{,}104$$

Finally, let us work out the product shown on the above Abacus photo. The simplest approach is the following-

$$36x72 = 36 \cdot 70 + 72 = 2520 + 72 = 2{,}592$$

This number can also be determined equally well by the operation-

$$36x72 = 2 \cdot 36^2 = 2 \cdot 9 \cdot 144 = 9 \cdot 288 = 2880 - 288 = 2{,}592$$

Feb.2012