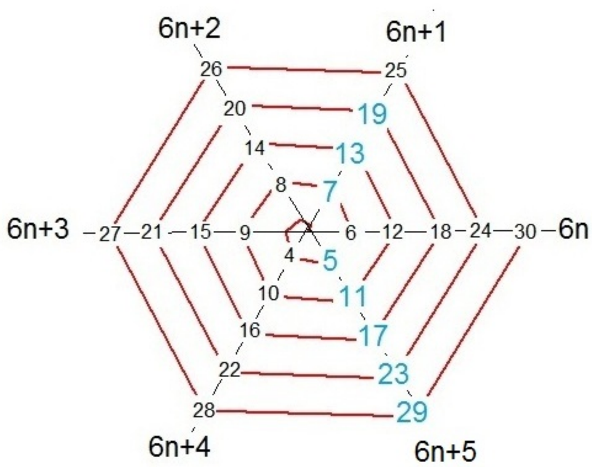
# EVEN AND ODD NUMBERS, PRIMES,TWIN PRIMES, COMPOSITES AND SUPER-COMPOSITES

All positive integers N=1,2,3,4,5,6,7.8,.. can be represented geometrically as points at the intersections of the hexagonal spiral Nexp(i$n\pi$/3) and the radial lines 6n, 6n+1, 6n+2, 6n+3, 6n+4,and 6n+5 in the complex plane as shown-

HEXAGONAL INTEGER SPIRAL




blue numbers indicate primes

We first came up with this representation in July of 2013 while in the process of analyzing the standard Ulam Spiral and its rather random distribution of primes(see http://www2.mae.ufl.edu/~uhk/MORPHING-ULAM.pdf). We note that all even numbers lie along 6n, 6n+2, and 6n+4. The odd integers lie along 6n+1, 6n+3, and 6n+5. Of particlar interest is the fact that-

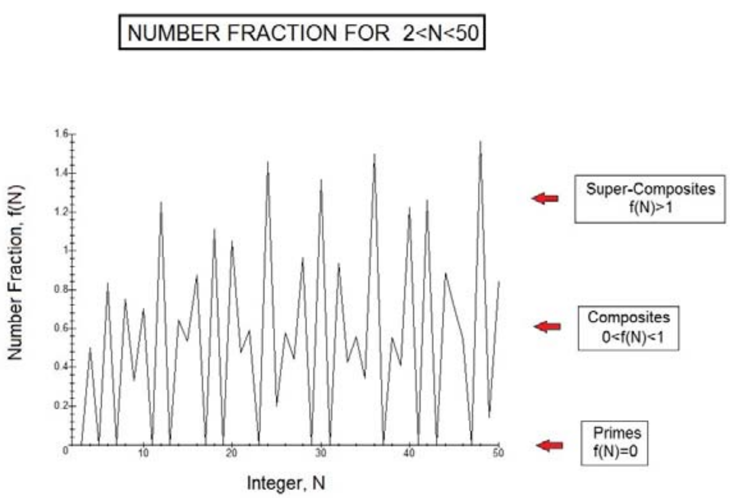
**A necessary but not sufficient condition for a number to be a prime (greater than three) is that it must lie along the radial lines 6n+1 or 6n-1 .**

The non-sufficient aspect of this observation is that there are also points along 6n±1 that are composite such as 25 ,35, etc. The nine digit long number N=34694089 =6(5782348)+1 lies along 6n+1 and is a prime number. The quickest way to determine along which radial line an integer falls is to perform the operation N mod(6). Thus the even number  N=69823043762 has N mod(6)=2 and so lies along the radial line 6n+2. When two primes p and q differ from each other by 2 they are referred to as twin primes. Some simple examples are [5,7],[11,13], and [41,43]. The mean value for each of these is an even number N=6n, n=1,2,3,etc.. So N=6(248297)=1489782 will produce a twin prime pair if both 1489783 and 1489781 are prime. They are, so we have the twin prime pair[1489781,1489783]. It is always the case that p and q for twin primes lie along opposing radial lines 6n+1 and 6n-1.

So far we have not made a distinction between composite and super-composite numbers other than to note the trivial point that all composites, unlike primes, are divisible by more than just 1 and N. We now discuss this distinction by introducing another new concept, namely the Number Fraction defined as-

$$f(N) = \frac{\sigma(N) - N - 1}{N}$$

, where σ(N) is the well known divisor function of N as it appears in number theory. We firtst came up with this point function back in Septemver of 2012 as discussed in the article [http://www2.mae.ufl.edu/~uhk/NUMBER-FRACTION.pdf](http://www2.mae.ufl.edu/~uhk/NUMBER-FRACTION.pdf) . In designing this function I made f(N)=0 for all primes with f(N) greater than zero for composites and super-composites. Also the N in the denominator was placed there to keep the value of f(N) from becoming too fast with increasing N. A graph of f(N) over the range 2<N<50 looks as follows-



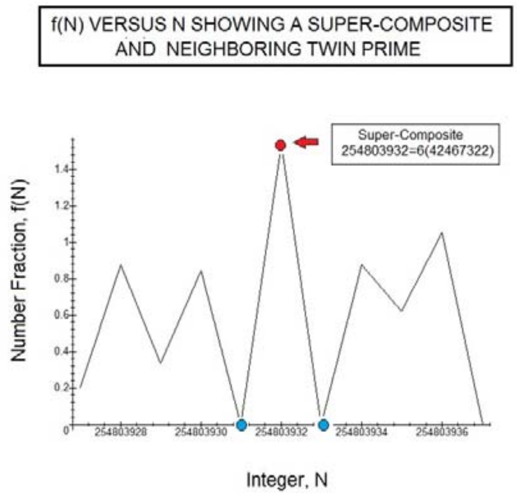

We see at once the primes 3,5,7,11,13,17,19,23,29,31,37,41,43,and 47 which all have f(N)-0. Standard composites are found in the range 0<f(N)<1. I have termed those f(N)>1 as super-composites as they have many more devisors than their neighbors. Five of the larger super-composites shown in the graph are 12, 24,30 36 and 48.Writing these out as their prime product forms yields-

| Super-Composite ,N | Ifactor |
|---|---|
| 12 | $(2^2)3$ |
| 24 | $(2^3)3$ |
| 30 | 2·3·5 |
| 36 | $(2^2)(3^2)$ |
| 48 | $(2^4)3$ |

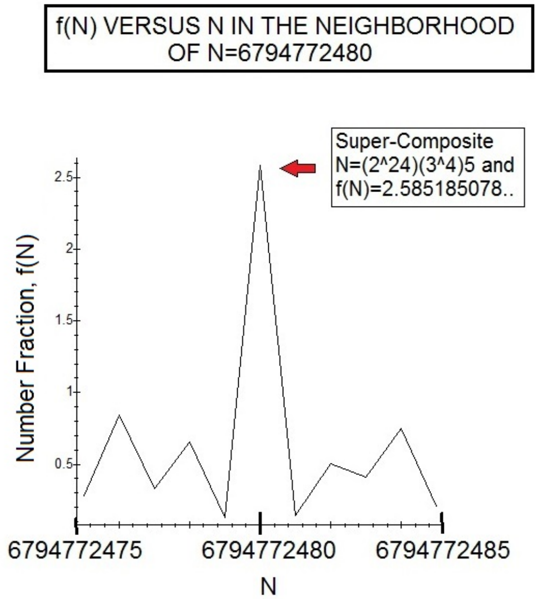
It is seen that these numbers only involve products of the lowest primes where the power of the n-1 prime is generally greater than the power of the nth prime. This means that we can call those integers where

$$N=2^a\cdot 3^b\cdot 5^c\cdot\ldots \text{ with } a>b>c$$

super-composites. Here is a plot about a super-composite N=254803932-




Here the prime product breakdown reads $N=2^2\cdot 3^2\cdot 191\cdot 37047$ . Even more distinct super-composites are found when the breakdown of a super-composite has a form not involving higher primes . For example $N=2^{24}\cdot 3^4\cdot 5^1=6794772480$ produces the following f(N) versus N graph-




We see a sharp peak at the super=composite with the immediate neighbors being very small but not equal to zero and hence not primes. Such small f(N) typically indicate semi-primes such that N=pq or tri-primes N=pqr. For 6794772481 we get 7(090681783) and for 6794772479 we have 11(31)(19926019).

One can factor any semi-prime N=pq by use of the identity Nf(N) =σ(N)-N-1=p+q. This produces-

$$[p,q]=\left[\frac{Nf(N)}{2}\right]\mp\sqrt{\left[\frac{Nf(N)}{2}\right]^2-N}$$

Fortunately σ(N) is known to at least 40 places in most advanced computer programs so that any semi-prime of forty places or less is readily factored by this formula. For an example, the Fermat number $N=2^{32}+1=4294967297$ has σ(N)=429538800. Thus we have

4294967297=641 x 6700417

Leonard Euler struggled for months to get this answer which we now generate in a split second. The key to breaking semi-primes of 100 digit length or so, as used in public cryptography, will be to find a way to quickly generate σ(N)s to that number of places.

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