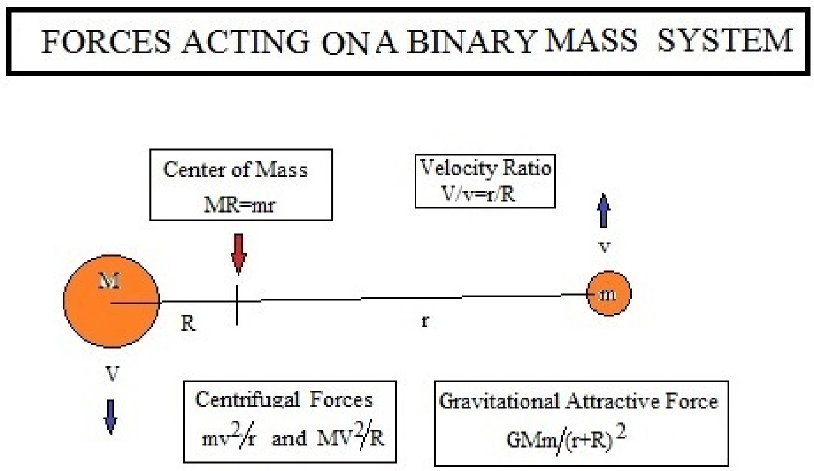
PROPERTIES OF BINARY MASS SYSTEMS

A binary mass system consists of two spherical masses M and m rotating with constant angular velocity ω about their common center of mass known as the barypoint. The masses assume an equilibrium position where their attractive gravitational force balances the centrifugal forces.The distances to the barypoint from the centers of the two spherical masses are taken as D and d. A schematic of such a binary set-up follows-



This arrangement can be used to describe a variety of different binary mass systems subjected to a mutual gravitational attraction. These systems include binary stars, a planet revolving about a central star, and satellites in orbit about a planet.

According to Newton thr gravitational attractive force between the two masses m and M is-

$$F_G = \frac{GMm}{(d+D)^2}$$

, with G being the universal gravitational constant equal to $6.672\text{x}10^{-11}$ Nm^2/kg^2 . Let us assume that these masses follow pure circular paths (ie-zero eccentricity) around their common center of mass. In that case the outward radial centrifugal forces are –

$$F_M = MD\omega^2 \quad and \quad F_m = md\omega^2$$

Since the sum of F_M and F_m must equal zero, we have that -

$$md=MD$$

This defines the location of the barypoint. The orbit period will be $\tau=2\pi/\omega$.
Balancing the gravitational force with the centrifugal forces we arrive at the identities-

$$\tau^2 = \frac{4\pi^2}{G}[\frac{D(d+D)^2}{m}] = \frac{4\pi^2}{G}[\frac{d(d+D)^2}{M}]$$

Plugging in for D, we get-

$$\tau^2 = \frac{4\pi^2}{GM}(1+\frac{m}{M})^2 d^3$$

This equation is a special form of Kepler's Third Law which says the square of an orbital period is proportional to the cube of radial distance from the center of the smaller mass m to the mass center of the binary. Since the quantities d, D, and τ are measurable quantities for most binary mass systems, we can also write this last equation as-

$$M = (\frac{4\pi^2 d}{G\tau^2})(d+D)^2$$

which yields the value of the larger mass. The smaller mass follows from m=MD/d.
Combining m and M, we get that the total mass M_{Total} of the binary system is given by-

$$M_{Total} = (M+m) = \frac{4\pi^2}{G\tau^2}(d+D)^3$$

Thus the total mass of a binary star may be determined knowing only its rotation period τ, the total distance between the star centers d+D, and the value of the universal gravitational constant G. Some changes will occur in this result if the paths of the components are ellipses and the view from earth does not lie in the binary orbit plane.

For the special case of an equal mass binary where m=M and d=D, we find-

$$\tau^2 = \frac{32\pi^2}{GM_{Total}} d^3.$$

The other extreme occurs when M>>m such as when we consider the Earth-Moon system as first done mathematically by Issac Newton. In this case the barypoint will be located very close to the Earth's center since D<<d. Also we have M>>m. These approximations allow us to state, for example, that the distance d to the moon equals-

$$d = \sqrt[3]{\frac{GM\tau^2}{4\pi^2}}$$

But one also has from the gravitational law that $GM=gR^2$ for a 1kg test mass placed at the earth's surface at r=R. We thus have the basic result-

$$d = \sqrt[3]{\frac{g(R\tau)^2}{4\pi^2}} = \left[\frac{9.81m/s^2(6371km27.3days)^2}{39.4784}\right]^{1/3} = 382{,}847km$$

This ia a pretty good approximation when compared to the laser determined value of 385,000 km=238,900 miles. Since all electromagnetic waves travel at 186 thousand miles per second in vacuum, a laser pulse sent from Earth bouncing of off a corner reflector on the moon and then returning to Earth will take about 2.57 seconds.. So one can say the Moon lies about 1 light second away from Earth. Astronomers have measured the average distance from the Earth to the Sun to be 93 million miles=1AU. The distance corresponds to 93,000,000/186,000=8.33 light minutes. The nearest star to our solar system is Proxima Centauri at 4.24 light years away. Our Milky Way Nebula is estimated to have a diameter of D=150,000 light years and a center thickness of about W=40,000 light years and contains about N=200 billion stars. Assuming a uniform star distribution, this means each star is located on average 16 light years from its nearest neighbor. Since intelligent life can be expected to develop on planets about these stars for only a very small fraction of their numbers, one can make the estimate that the nearest intelligent life (if it exists) must be at least 2000 light year way from us. It is very unlikely that man will ever be able to reach such distances physically or even just holding electronic communications. Just imagine trying to communicate with someone where a single round trip for a question and answer will take 4000 years or more. This answers Enrico Fermi's question of " Where are They?" when asked about aliens from outer space.

To determine the distance to the sun one can use some trigonometry using the Earth, Venus, and Sun as a triangle. Using the radar measured distance between Earth and Venus for set angles of the triangle, showsa Sun-Earth distance of 1 Astronomical Unit(1AU)=93×10^6miles. Once the distance between Earth and Sun is known, it is a straight forward procedure to show that the sun's mass equals $M=1.99\times10^{30}$kg. The Earth-Moon binary system yields the Earth's mass as 5.97×10^{24}kg as first shown by Cavendish.

A near earth satellite moving in a circular orbit about the earth has d=R=6371km, $D/d\approx0$, and $GM=R^2g$. So the above τ formula reads-

$$\tau = 2\pi\sqrt{\frac{R}{g}} = 1hr25\min$$

I remember as a twenty year old physics major back on October 4th, 1956 getting up at 4:30am in the morning waiting for the first earth satellite Sputnik to cross the sky. It was quite a site seeing the reflection of this first man-made satellite as it streaked across the sky. Its period was τ=96 minutes since it had a longer elliptic trajectory with an apogee of 939km and perigee of 215km.

On July 4th NASA placed its Juno satellite into a highly elliptic orbit about Jupiter. Had it been put into a pure circular one near the planrt's surface at R=43441 miles , one would have from Kepler's Third Law, that-

$$\frac{\tau_{Jupiter}}{\tau_{Earth}} = \left(\frac{43441miles}{3959miles}\right)^{3/2} = 36.35$$

This implies a τ=2.15 day orbit instead of the final highly eccentric orbit of τ=14 days to be assumed by Juno after an orbit correction this fall.

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