CALCULATING IN BINARY

Modern communications be it via computers, telephone, or television are accomplished via binary signals of 1 (on) and 0 (off). A concatenation in time of such zeros and ones can be made to transmit meaningful information be it as speech, music, computer and CD files, or pictures in an undistorted manner not possible with analogue signals. The mathematics behind this type of digital communication is the binary number system in which the decimal digits of 1, 2, 3, 4, 5, 6, 7, 8, and 9 are re-written as 1, 10, 11, 100, 101, 110, 111, 1000, and 1001, respectively. The binary numbers are expressed as powers of two, so that it is a good idea for individuals working with this number system to be familiar with the first ten or so powers of two as shown in the following table-

n	0	1	2	3	4	5	6	7	8	9	10
2^n	1	2	4	8	16	32	64	128	256	512	1024

The binary form off the number 2^n will be designated as one followed by n zeros. Thus 256 will read 100000000 and 275=256+16+2+1 will read 100010011. The basic addition and subtraction rules in binary are-

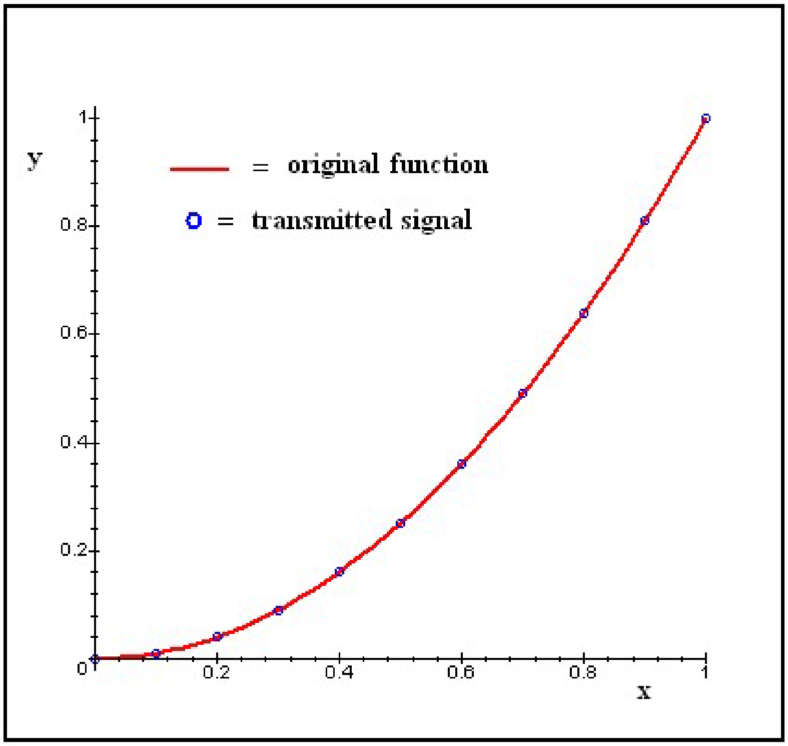
$$1+1=10, 1+0=1, 0+0=0, 1-0=1, 1-1=0, \textit{ and } 0-1=1 \textit{ borrow } 1$$

To double a binary number one just adds a 0 to the end of the number. Thus 22=10110 becomes 44=101100 and 88=1011000. To half it just take away a 0 at the end. Thus 50/2=110010/10=11001. Also half of 33 becomes 10000.1 where the part of the binary number to the right of the period represents inverse powers of two. If you subtract 1 from 2^n the binary number will be a series of n-1 ones with no zeros. Thus 63 in binary is 11111 and 511 will be 11111111.

With the above information one is now ready to transmit in binary. Lets begin with a simple parabola defined as y=x^2 in the range 0≤x≤1. Let us sample this at intervals of Δx=1/10 getting 100y=0, 1, 4, 9, 16, 25, 36, 49, 64 , 81, 100. One can then transmit the signal-

10x	0	1	2	3	4	5	6	7	8	9	10
100y	0	1	4	9	16	25	36	49	64	81	100

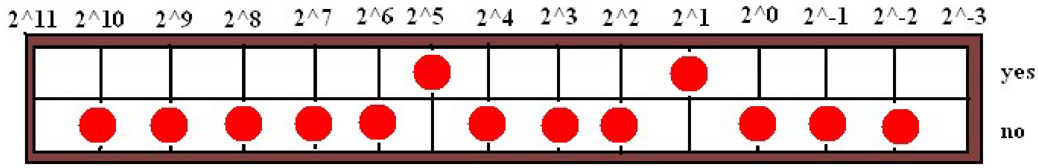
in binary form as a concatenation of ones and zeros. At the receiver the signal can be reconstructed to obtain the graph-



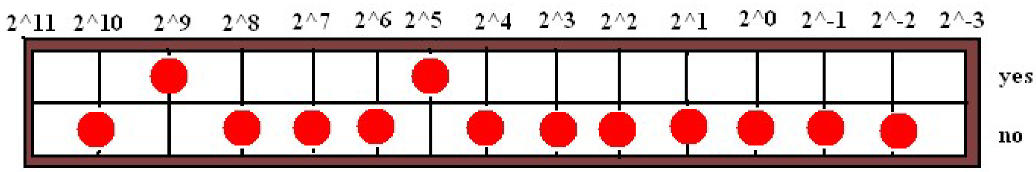
The smaller Δx becomes the more closely the transmitted signal matches the original. Digitized high fidelity music is typically sampled at a frequency of f= 96Khz so that the time interval between sampling points is about 10 millionth of a second. Such short intervals are required because the human ear can hear frequencies up to about 20Khz.

As we have already seen, binary requires just two symbols to express a number rather than the ten required in decimal. This allows one to introduce a greatly simplified version of the classical abacus (or Soroban) involving just a single bead per column to carry out addition, subtraction, multiplication , and division. The binary abacus will look like this-

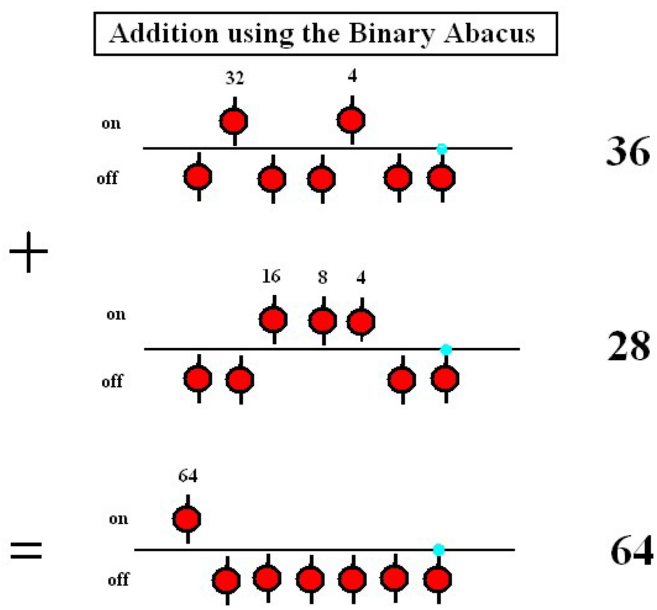
BINARY ABACUS SHOWING THE NUMBER THIRTY FOUR



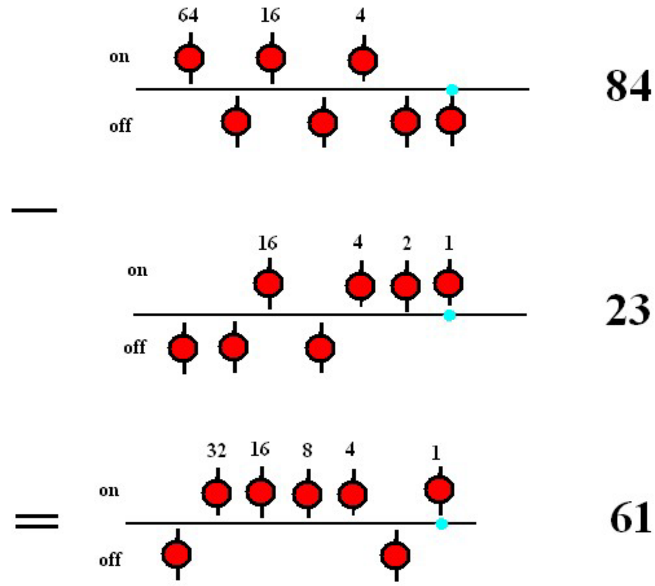
BEAD POSITION FOR THE PRODUCT 34x16 =544



The manipulation of the beads just follows the very simple rules described above for binary numbers. In the above example of multiplying 34 by 16 , one just needs to move the original pattern by four columns to the left. Here are two examples showing addition and subtraction operations on a binary abacus-

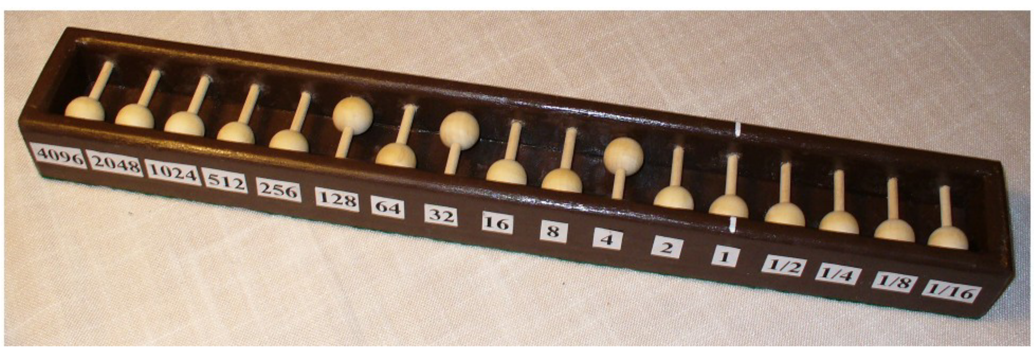


Subtraction using Binary Abacus



In the following picture I show you a Binary-Decimal Converter which I recently built in my workshop. It shows the number 164→10100100. The device can be used to carry out the mathematical operations of addition, subtraction, multiplication, and division over the range 2^{-4} to 2^{+12}.

BINARY ABACUS AND DECIMAL-BINARY CONVERTER



Showing Number 164 = 10100100

Before 2000 stock and bond quotes had fractional endings such ½, ¼, 1/8, 1/16, 1/32,and 1/64. Such endings are well suited for binary representation. Thus a stock selling at $19+1/8 and a bond selling at $100+1/32 would have a total cost written in binary as-

10011.001+ 1100100.00001=1110111.00101 or (119+5/32 in decimal)

Doubling and halving problems are also well treated in binary language. Take the case of adding the series-

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots \textit{ in binary this reads } 0+.11111111111....$$

But we recognize that this in decimal means 1+(2-1)=2. Thus, this geometric series sums to 2 (recall Zeno's paradox).

Another problem involving binary arithmetic deals with the following and probably apocryphal story of a Persian king and a poor but clever suitor who wanted to marry the king's daughter. As a dowry the suitor ask the king to supply him with all the grains of rice accumulated by placing one grain on the first square of a chess board, then two on the next square and so on doubling things until the 64^{th} square is reached. The king readily agreed to the suitor's request, thinking he got the better of the deal. Was the king fooled? Here is the answer. The total number of rice grains is-

N=1+2+4+8+16+32+ …..+2^63 in decimal

or in binary it's a row of 63 ones-

N=111111111….11111

But this last number is just equivalent to 2^64 -1 when converted into decimal. Hence-

N=2^64-1=4^32-1=16^16-1=256^8-1=65536^4-1
= 4294967296^2-1=18,446,744,073,709,551,615

That is , about 18 quintillion grains of rice. Since each pound of rice contains about 29,000 grains, the number of grains N will amount to about 318 billion tons compared to the presently estimated 400 million tons of yearly rice production in the world.