In several recent notes we have shown how one can construct 2D curves from straight lines, connected to neighboring lines at fixed angle, by use of simple genetic codes. We want here to look at this problem in still more detail for the special case where each line has unit length and is connected to its neighbor in one of three possible ways, namely,

\[ + \text{ for a } \pi/2 \text{ radian turn to the left} \]

\[ 0 \text{ for no change in direction} \]

\[ - \text{ for } \pi/2 \text{ radian turn to the right} \]

Consider first the twelve sided figure defined by the code-

\[[+ + - + + - + + - + + -] \]

As shown in the accompanying figure it produces a Swiss Cross whose basic genetic code generator is \([+ + -]\) repeated four times-

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**Swiss Cross**

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+ + - + + - + + - + + - + + -
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basic code \([- + +]\)
In generating figures such as this we will move along the curve in a counterclockwise manner. Note here that this basic code contains one – bend and two + bends yielding a net of one positive bend. To generate a closed figure from this three element code one must require the repetition of the basic three digit genetic code a total of four times ( that is \(4(\pi/2)=2\pi\) ). Also one needs to verify that the resultant curve is indeed closed by showing that the sum of all unit edges in the x and y directions add up to zero when taking the appropriate sign convention into account. In this example we have the sum of the xs equal to \(-1-1-1+1+1+1=0\) and the sum of the ys equal to \(1-1-1+1+1+1=0\).

Another example of a genetic code is the multiple pulse function \([+ - - +]\). A repetition of this code n times will produce n rectangular pulses as shown:

- **Rectangular Pulse Function**

( showing operation \(+ - - + - - + - - + - + - +\) )

- **Basic Genetic Code** \([+ - - +]\)

Since the sum of the plus and minus signs are equal in this case, the generated curve should remain open and eventually approach \(\pm\) infinity. Non-closed curves will generally be generated by codes where the number of + and – signs are equal while those with unequal numbers tend to produce closed curves upon multiple applications of the code. There are exceptions to this rule such as the code \([+++-+++]\) which does not have the same starting and end points yet has a net of \(2\pi\) radian turn.

Look next at the more complicated nine element genetic code-

\([-0++00+0-]\)

What type of curve does it represent? The code by itself produces an open ended letter L but when repeated three more times produces the standard swastika which represents a closed curve. This observation follows from the fact that the basic code has 3 + elements and the 2 – elements showing that a closed curve can possibly be
obtained by a total of four repetitions. To guarantee that this curve will be closed one needs to check that the sum of the edges in both the x and y directions add up to a net value of zero. This is the case for the swastika.

Genetic codes say nothing about the orientation of the figure. Also magnification or reduction of the figure size is easily accomplished by changing the unit length of the border elements.

The number of repetitions of a genetic code needed to produce a possible closed curve is determined by the difference in the number of + and – signs in the code. Thus-

\[ [+0-+-+-0+] \]

should produce a closed figure with four repetitions. That is indeed the case as shown in the following, filled in black, curve-

NINE ELEMENT GENETIC CODE

[ - ++ - ++ - + ]

The genetic code-

[ - ++ - ++ - + ]
has five + signs and three - signs. Thus a closed figure is likely to result from the string -

\[
[-++-++-++-++-++-++]
\]

This is indeed what happens as shown-

![Double Cross](image)

Genetic Code: [-++-++]

Another example of a code producing a closed curve is [+ - - +] applied four times. It produces the checkerboard pattern shown-
What is remarkable about these types of genetic code representations is that intricate figures can be constructed from very simple codes. Let us see what happens when we pick the symbols +, 0, and – in a random manner. Using a dice and letting 1 or 4 represent +, 2 or 5 represent –, and 3 or 6 represent 0, we have generated the following 12 element genetic code-

\[ [ + 0 - + - 0 0 + + - ] \]

There are four elements of each of the three symbols. Hence we can say that it will represent an open curve no matter how many repetitions of the basic code one applies. A graph of this code showing its start and end points follows-
Had we stopped the code generation after ten throws of the dice then the last two – signs would be missing and one would have 4 + signs and two – signs in the code. This implies the existence of a closed curve upon using the code twice as shown-
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