WHAT IS COMPOUND INTEREST?

One of the basic tenets of capitalism is to try to maximize the return on one's capital. One way to obtain such a gain is to loan out the capital C_0 at a positive rate of interest i . This is known as simple interest and is something the Mafia might use charging exorbitant interest over a short period of time. For the case of simple interest the interest is paid as a single payment at the end of a specified holding period and the resultant new sum $C_0(1+i)$ is not re-invested. A much better scheme for maximizing returns (and first used by the 14th and 15th century Renaissance bankers of Florence) is to keep investing the yearly sum over and over. That is, one is dealing with interest on interest, a practice condemned as usury by the early Christian church and still not approved of by many people around the world to this day. This practice is known as compounding and produces the result after n years of-

$$C = C_0(1+i)^n$$

, when the interest i is paid once a year. If instead one receives half the interest every six months, the capital will grow as

$$C = C_0(1+\frac{i}{2})^{2n}$$

Continuing on to a quarterly payment results in-

$$C = C_0(1+\frac{i}{4})^{4n}$$

Extending the payments to k yearly equal sub-intervals results in-

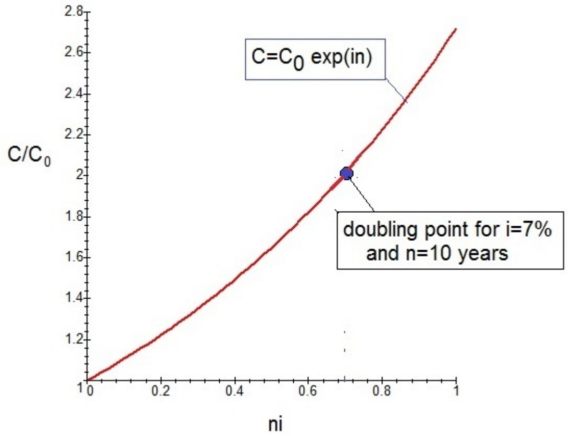
$$C = C_0(1+\frac{i}{k})^{nk}$$

On taking the limit of k going to infinity we get-

$$C/C_0 = \frac{\lim}{k\to\infty}\ (1+\frac{i}{k})^{kn} = \exp(ni) = 1+ni+\frac{(ni)^2}{2!}+\frac{(ni)^3}{3!}\_\ldots$$

This last case is the largest possible return and constitutes what is known as the return on capital for <u>continuous compounding</u>. A graph of C/Co versus ni follows-

CONTINUOUSLY COMPOUNDED RETURN VERSUS THE PRODUCT OF INTEREST RATE AND YEARS OF HOLDING



The increase in C/C_0 is seen to follow an exponential curve and thus can become quite large in only a few years. The difference between continuous compounding of C/C_0 at 7% for ten years and that given on a one year interest payment schedule can be considerable. This difference is often used to advantage by bankers to make money as well as harvesting large returns on car, home, credit card, and student loans. For the case of i=0.07 and n=10 we get-

$$\{ \exp(0.7)=2.0137\}>\{(1+0.07)^{10}=1.96715\}$$

Since common interest rates typically lie considerably below 10%, one sees that the capital will double when-

$$ni = \ln(2) = 0.693147 \approx 0.7$$

So we have the well known rule of thumb that capital will double in about ten years when continually compounded at an interest rate of seven percent. At i=0.035(3.5%) a doubling of C_o will occur in about 20 years;

It is also possible to arrive at the continuous compound interest result via the first order differential equation-

$$\frac{dC}{dt} = iC \quad subject \quad to \quad C(0) = C_0$$

It solves as –

$$C(t) = C_0 \exp(it)$$

Here t is the time expressed in years n. It was while studying this solution that the famous Swiss mathematician J.Bernuolli first came up with the universal constant e=2.71828…=exp(1) in 1683.

What has been neglected in the above discussion on the return of capital with a compound interest rate i is the factor of inflation f and the tax rate for a bracket b on any gains. To discuss these effects we look at the case of a single annual compounding (k=1). Here we have a gain on paper , after taxes, of $C_0i(1-b)$. But after inflation f is taken into account, this gain is further reduced to

$$C_0[i(1-b)](1-f)\}$$

In addition one should not forget that inflation f also reduces the net worth of C_0 by $C_0(1-f)$ anually. This implies that the true gain on C_0 in one year will be-

$$\text{True Gain}=C_0\{-f +[i(1-b)](1-f)\}$$

This result shows that there will be a true gain only if $f<[i(1-b)](1-f)$. As an example consider the case where C_0=3 million dollars is put into a money market fund offering i=2% at an annual inflation rate of f=2%, and a tax bracket of b=30%. After one year this produces a gain of-

$$\text{True Gain}=3\text{x}10^6\{-0.02+[0.02(1-0.3)](1-0.02)\}= -\$18,840$$

That is, one actually has a real loss of nearly 19 thousand dollars in one year. So clearly it is not a good investment! To stay ahead of the game will require that one obtains at least as much interest income so as to overcome any inflation losses. Generalized this means that return on capital must exceed the losses in dollar value due to inflation. Typically this will not be accomplished through bank savings and other interest bearing means such as bonds but rather requires investment in stocks and real estate. There the returns can be high, easily overcoming inflation losses. However, such investments also will introduce greater risk , especially when carried out on margin and after having reached historic highs.

The Federal Reserve's policy of keeping inflation rates at an average of 2% (f=0.02) means that if you stuff your dollars under a mattress and not invest them they will loose half their value in 35 years. Indeed, in the last hundred years the Federal Reserve has managed to debauch the dollar's intrinsic value by a factor of about $C_0/C(100)=\exp[0.02(100)] =7.39$. The actions of the Federal Reserve Chairman Ben Bernanke during the last decade has made things even worse by the pumping of some three trillion fiat dollars into the financial system. It has not really improved the well being of the average American family but has succeeded in driving up the price of stocks, real-estate, art and other collectibles to all time highs, acerbating the gap between the rich and poor in this country. The present members of the Federal Reserve are now finding themselves in a major dilemma. They know that inflation will accelerate beyond their 2% guideline if they don't raise interest rates fast enough but realize that if they do this inflated stock and real estate markets could collapse. The effect of the new Trump tariff policy against certain foreign countries is expected to put additional upward pressure on inflation and interest rates and make it more difficult for the US to keep up interest payments on its ever increasing national debt.

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