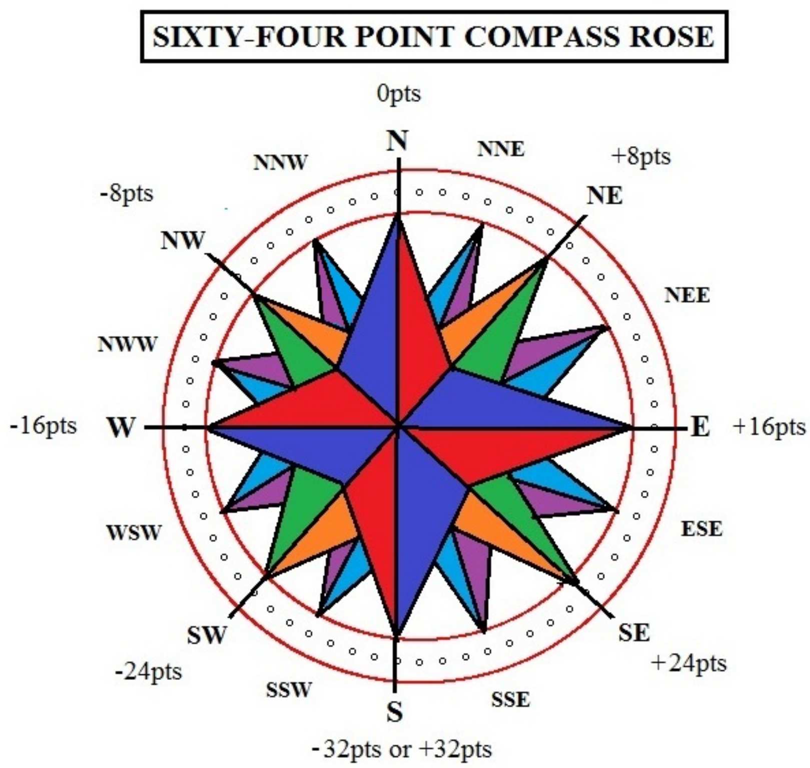
CONCATENATION OF STRAIGHT LINE SEGMENTS INTO OPEN AND CLOSED ENDED 2D CURVES

In the early days (prior to accurate optical and especially laser methods) the standard way to define a parcel of land was to designate the length of its straight-line sides by means of the number of paces it took to go from one end to the other plus giving the angle along which the straight line lay. So if you owned a piece of farmland in the 18th century it might have had the designation-

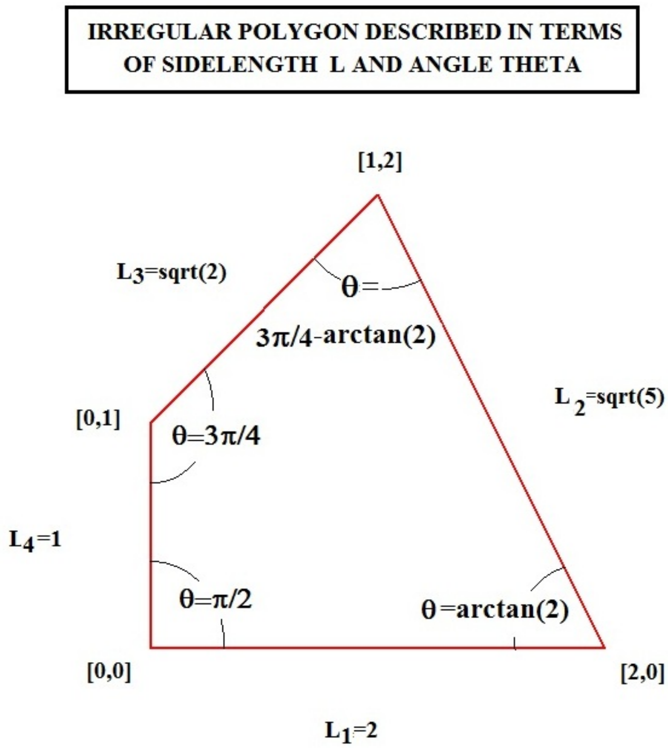
[L,θ]=[200, E]..[100sqrt(5),-4.723 pts W of N]..[100sqrt(20,SW]..[100,S]

Here E, N, W, and S represent the four cardinal directions on a compass. NNW indicates a direction north by north-west or -22.5deg from the north direction N. That is -4 points from N using a 64 point compass. A point is separated on a compass by $\Delta\theta = 5.625$ deg for a total of sixty four per 360deg. East E would be +16points from north (N) and south S either -32 or +32 points from north N. This point measuring system is made clear by the following compass rose-



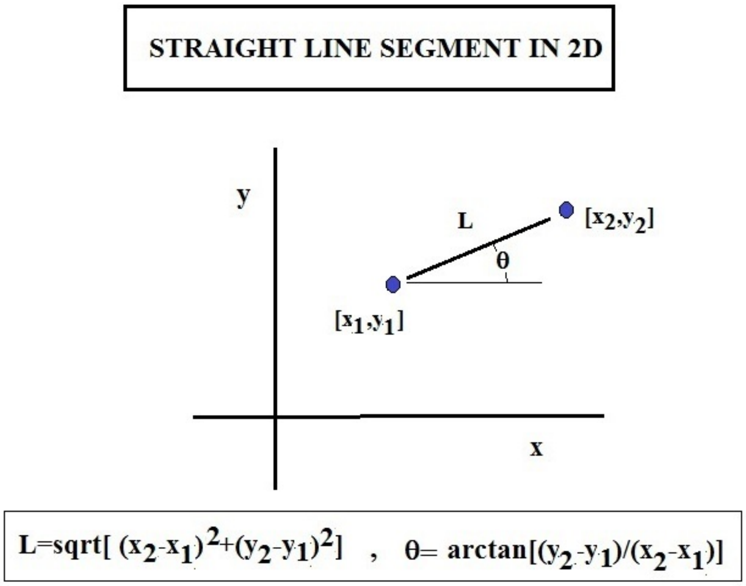
small circles are separated from each other by π/32 radians

For most land measurements this yielded sufficient accuracy since the corners of the irregular polygon structure where typically recognizable entities such as a large tree or rock. Here is a schematic of an irregular polygon defined by the above measurements-



By subdividing the area into sub-areas it is easy to show that the total area is exactly $2.5\text{x}100^2$ ft^2=0.574 acres. Recall that one acre equals 43,560 ft^2 or 1/640 mile^2. As expected, the sum of the interior angles of thefour-corner piece of land is 2π radians. During the Homestead Act in American history farmers were offered free land in many of the westerns and central states provided they would farm the land successfully for five years. The land parcels were large ranging fron ¼ sections (160 acres) to full sections(640 acres). This has made many of the offspring of these impoverished immigrants from Europe super-millionaires.

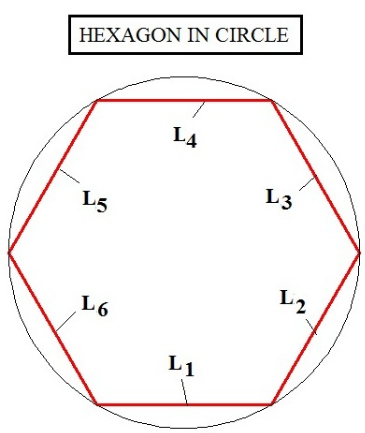
It is our purpose here to show how one can construct both open and closed 2D curves using a concatenation of straight line segments as discussed above for land measure. We begin with some very simple 2D curves. Each line increment is designated by just its length and the angle θ relative to a chosen axis such as the x axis. Here is a schematic of one of these elements-



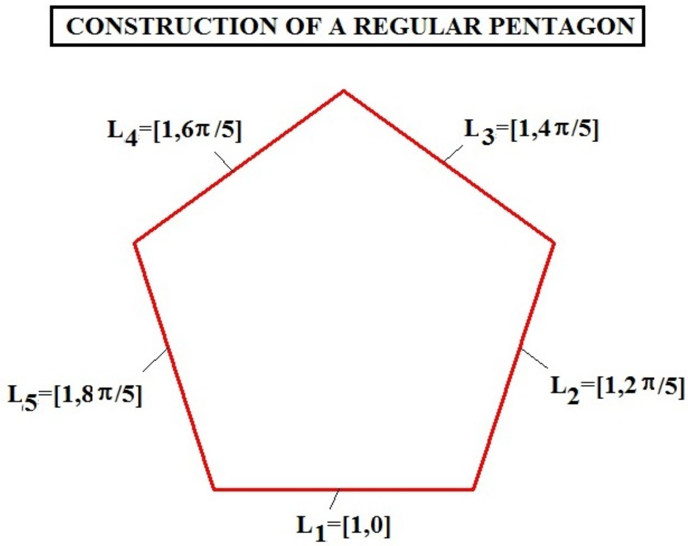
Let us look at the curve constructed by six-straight line segments of length L=1 each and an orientation given by the angle they make relative to the x axis. The configuration reads-

$$[L,\theta]=[1,0]..[1,\frac{\pi}{3}]..[1,\frac{2\pi}{3}],,[1,\pi]..[1,\frac{4\pi}{3}]..[1,\frac{5\pi}{3}]$$

It is very simple to construct this 2D curve by use of a protractor and compass. All corners of the 2D curve must lie on a radius one circle. The first element is a horizontal line segment of length L=1 matching the radius . It is symmetric about the y axis. Drawing all straight line segments we get the closed hexagon shown-



Any other n sided regular polygon can also be constructed by placing n equally spaced points around the periphery of a constant radius circle and then connecting neighboring points by straight lines. For a regular pentagon on has five straight line elements the first and second of which read [L,0] followed by [L,2π/5]. A picture of such a regular pentagon follows-

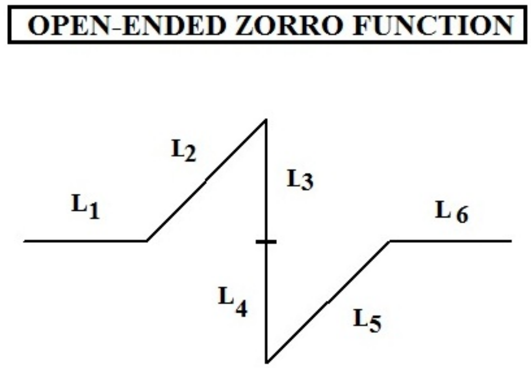


Note that one can scale this figure by changing L and can produce a counterclockwise rotation of angle θ by replacing the angle 2nπ/5 by 2nπ/5+θ. To construct a circle one simply lets the constant spacing between equally spaced points an a given guide circle go toward zero. A computer sketch of a fifty-sided regular polygon will look essentially like a circle.

Open ended 2D curves may also be constructed by this technique. Take the case of-

$$[L,\theta]=[1,0]..[\sqrt{2},\frac{\pi}{4}]..[1,\frac{-\pi}{2}]..[1,\frac{-\pi}{2}]..[1,\frac{\pi}{4}]-[1,0]$$

This produces the following-

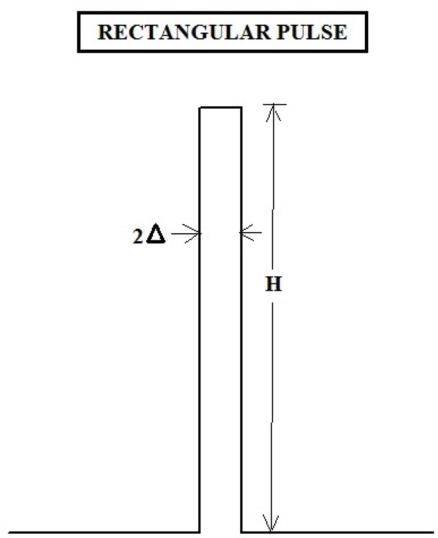


I call it the Zorro Function because of the slanted letter z.

A rectangular pulse function can be produced by the five segment arrangement-

$$[\, L,\theta]=[1-\Delta,0]..[H,\pi/2]..[2\Delta,0]..[H,-\pi/2]..[1-\Delta,0]$$

It looks like this-

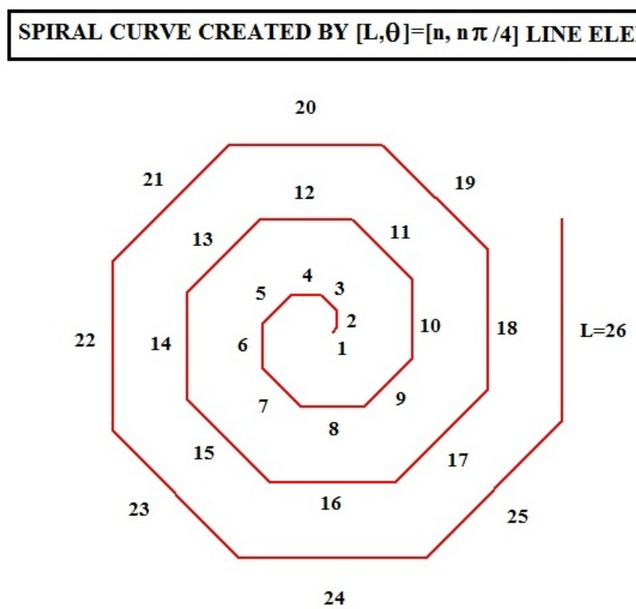


This function becomes the Dirac Delta Function when Δ is allowed to approach zero but the product $2\Delta H$ is kept equal to unity.

To construct a spiral using only straight line increments one can try elements defined by-

$$[L,\theta]=[n, \pi n/4] \quad \text{with} \quad n=1, 2, 3, 4,\ldots$$

Here the increment length grows by integer values between neighboring elements and the angle shifts by 45 deg per neighboring element. A graph of the resultant curve looks like this-



Curves resembling this spiral have been used by us in earlier articles to distinguish prime from composite numbers (type in U.H.Kurzweg Integer Spirals in the Google search engine to find these). Its shape is not very far removed from a standard Archimedes Spiral.

One can construct an infinite number of other straight line concatenations using the $\{L,\theta\}$ designation. It is also possible to go to 3D curves, but there one often runs into the Bridges of Koenigsberg problem in that most 3D wire configurations cannot exist without needing to retrace several of its straight line edges. For example, it is impossible to construct a simple 3D cube by straight line elements without some retracement of one or more of its edges.

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