ALL PRIMES GREATER THAN THREE HAVE THE FORM 6n±1

About six years ago (see our MATHFUNC page for Sept.1, 2012) I came up with a new point function f(N)=[sigma(N)-N-1]/N, where sigma is the familiar sum of divisors function of number theory. The interesting property of f (N) is that it vanishes whenever N is a prime but never when it is a composite. So when plotting this function over different intervals of N we found that it vanished only when N=6N±1, provided N was five or greater. This fact led us to the conclusion that-

All primes greater than three have the form N=6n±1

We have not found any exceptions to this rule for any of the primes tried. So, for example, the large prime number-

N=119557439128494976242815312498993526059074537858375287516106625293811638810057700278312003

has N mod(6)=1 so that N=6n+1. Another prime is N= 479001599. It yields N mod(6)=5, meaning that it has the form N=6n-1. It is the purpose of this article to confirm the above observation by starting from scratch using only a regrouping of the first few integers in ascending order.

We begin with the sequence-

S={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21, 22,23, 24,25,26,27,28,29,30,31,…}

Next we drop all even terms except two and also drop all odd terms which are divisible by more than just one. Also dropping N=1, then produces the Sieve of Eratosthenes result-

P={2,3}+{5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97,…}

These represent all primes up through N=100. Next we introduce the new concept of breaking the terms in the second bracket of P into two sequences-

U={7,13,19,31,37,43,61,67,73,79,97} and V={5,11,17,23,29,41,47,53,59,71,83,89}

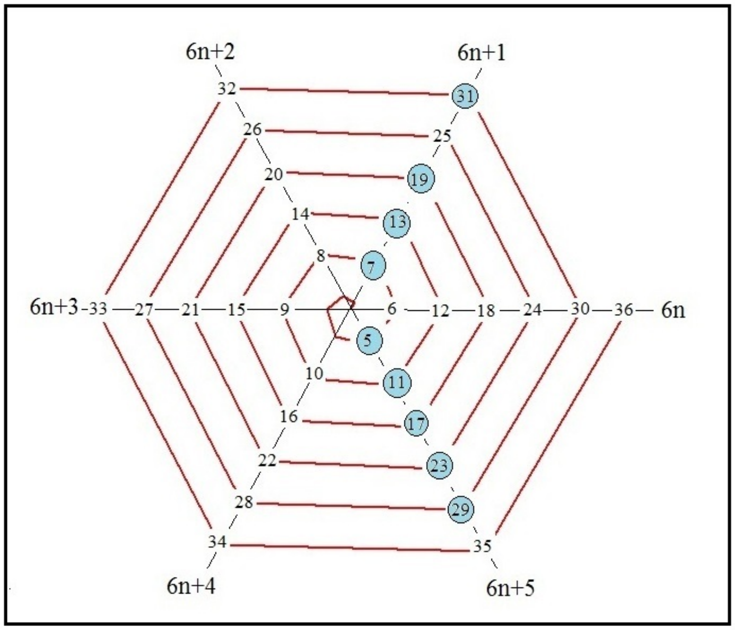
What you will notice is that all elements in the last two sequences differ from each other by factors of six. Thus, for example, 79-37=42=6(7) and 89-23=66=6(10). This means that we can reproduce all elements in sequence U by 7+6n and in sequence V by 5+6n . That is, any element in U has the property that a mod(6) operation produces 1 and for sequence V the elements have the property that a mod(6) operation yields 5 or its equivalent -1. This means that all elements in sequence U must equal 6n+1 and all elements in sequence V equal 6n-1. Our original observation is thus confirmed for any prime greater than N=3 since we are neglecting the first bracket {2,3]. We note, however, that some forms of 6n±1 are actually composites such as 25=6(4)+1 and 35=6(6)-1. So that we also have the alternate statement that-

A necessary <u>but not sufficient condition</u> for N to be prime is that it equals 6n±1

This still means that –

Any prime greater than three has the form 6n±1

A graphical representation for all primes five or greater can be displayed as the intersection points of the radial lines 6n+1 and 6n-1 and a hexagonal integer spiral defined as z=N exp(iπN/3). Here is the picture-



The primes are marked as blue circles. The graph shows that the lowest primes five or greater all fall along the radial lines 6n+1 or 6n+5. The latter is equivalent to 6(n+1)-1. The location of the prime 23 =6(4)-1 lies in the z plane at –

z=23{cos(π/3-isin(π/3)=23{(1/2)-i(sqrt(3)/2)}

The magnitude of z is just |z|=23. The gaps in primes along the radial lines 6n±1 seen for N=25 and 35 are composites which very often are just semi-primes N=pq. One also sees from the diagram that odd numbers of the form 6n±3 can never be prime. As |z| gets large the density of prime gaps along 6n±1 lines increases monotonically.

Composite values of N along 6n±1 are detected by dividing N by lower primes up to sqrt(N). If the resultant quotient is an integer then N is a composite. The gap at N=35 is composite since the primes 5 or 7 divide into it. For the number N=1243=6(207)+1 we have sqrt(N)=35.25. So we try dividing N by 7,11,13,17,19,23,29,31. Division by 11 produces the integer 113 and hence N=1243 is a composite with 1243=11 x 113. By use of a PC, the location of prime gaps along the radial lines 6n±1 becomes a simple matter. The computer program reads-

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for n from 1 to 50 do {n,6*n+1,isprime(6*n+1),6*n-1,isprime(6*n-1)}od;
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Those answers where isprime is false will yield a composite. Here is a table for the composites along the radial lines 6n+1 and 6n-1 for n=1 through 50-

n	6n+1	N	6n-1
4	25	6	35
8	49	11	65
9	55	13	77
14	85	16	95
15	91	20	119
19	115	21	125
20	121	24	143
22	133	26	155
24	145	27	161
28	169	31	185
29	175	34	203
31	187	35	209
34	205	36	215
36	217	37	221
39	235	41	245
41	247	46	275
42	253	48	287
48	289	50	299
49	295		
50	301		

Note that the last integer in these results is always odd . Also those values ending in five are obviously composite and the square or other integer powers of a number must also be composite. So N=4209875=6(701646)-1 is a composite. Also $N=2474329=(1573)^2$ =6(412388)+1 is also a composite. Composites lying along either 6n+1 or 6n-1 differ from each other by multiples of six.Thus 289-91=6(33) and 287-161=6(21)

A final observation concerns semi-primes N=pq. Here p=6n±1 with q=6m±1 or p=6n±1 with $q=6m\mp 1$. Expanding things then yield-

$$N=6\{6nm\pm(n+m)\}+1 \qquad \text{or} \qquad N=6\{6nm\pm(m-n)\}-1$$

Since the terms in the curly brackets are integers , we can state that for semi-primes we have either-

$$N=6k+1 \quad \text{or} \quad N=6j-1$$

, where k and j are integers. This last result allows one to state that all semi-primes with prime components greater than three must lie along the same two radial lines within a gap left between primes. Thus we have as an example the identity-

$$N=1928052461=34337 \times 56149$$

This may be rewritten as-

$$N=6(321342077)-1=[6(5723)-1\ ] \times [6(9358)+1] \qquad .$$

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