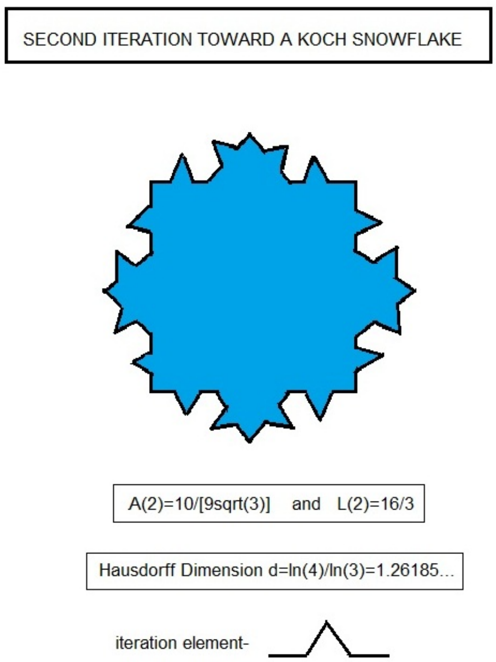
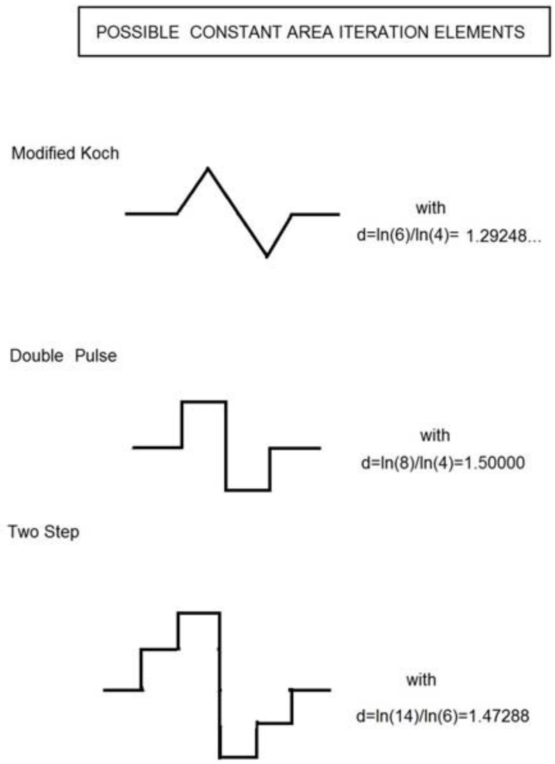
CONSTRUCTING FRACTAL CURVES FROM A UNIT SQUARE USING DIFFERENT ITERATIOIN ELEMENTS

The Swedish mathematician Helge von Koch (1870-1924) first showed that by iterating a equilateral triangle by dividing each side into three parts and then replacing the central portion of the edge by a smaller equilateral triangle, that , when this iteration procedure is repeated indefinitely, the result is a figure known as the Koch Snowflake. This figure has finite area but an infinite length boundary. The second iteration (generation) toward the snowflake looks as follows-



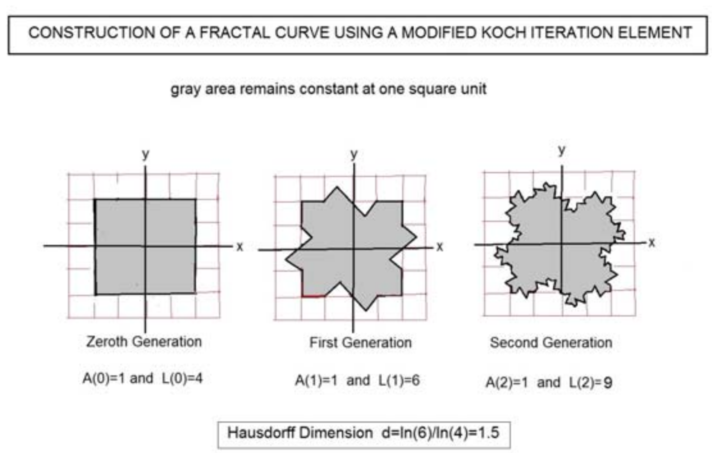
The infinite length boundary of the snowflake is referred to as a fractal curve since its edge construction make use of the same element but at progressively smaller size.

The boundary curve starts with a unit square and then develops through repeated iterations using ever smaller self-similar elements. We will consider only those elements which guarantee that the enclosed area of the fractal curve retains its value of one. Here is a list of three iteration elements (of an infinite possible number) we will employ-



We will show the results of the first few iterations of the above elements as they pertain to a unit square. The construction will be by use of a rectangular grid showing the zero, first and second generation toward the desired fractal curve.

We begin with the Modified Koch Element starting at the top left vertex of a unit square. We find the following-

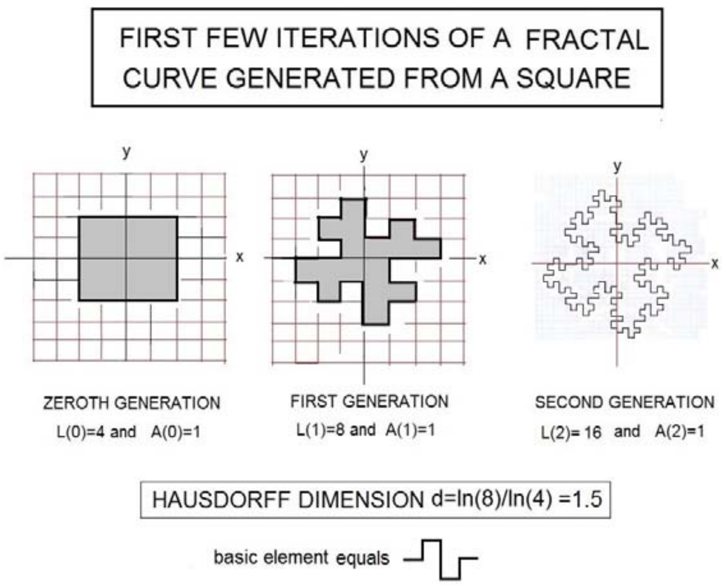


As seen the area contained by the fractal boundary curve stays at one unit but the curve length increases with the number of iterations. The nth generation has a curve length related to the previous generation by the formula L(n)=1.5 L(n-1). This is equivalent to saying that-

$$L(n)=4(3/2)^n$$

Thus as the iterations go to infinity we have a fractal curve of infinite length. Note that the factor 3/2 stems from the fraction 6 to 4 where 6 is the total element length and 4 the strait line distance between the endpoints of the element.

Consider next the curve generated by the Double Pulse element starting at the upper left corner of the square. Working out the first few iterations we get the following-

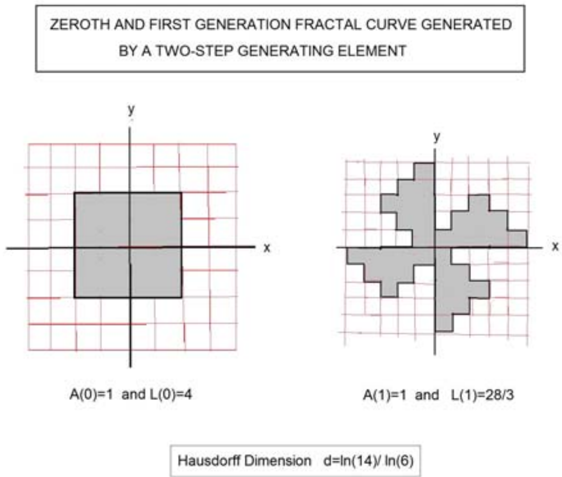


Here the area remains unchanged as one goes through the iterations while the curve length increases following the formula-

$$L(n)=4(2^n)$$

The 2 in this expression follows from the total element length of 8 compared to the straight line distance between its ends of 4.

Our third fractal curve will be generated using the Two-Step element already defined above. Construction of the zero and first iteration produces the following –



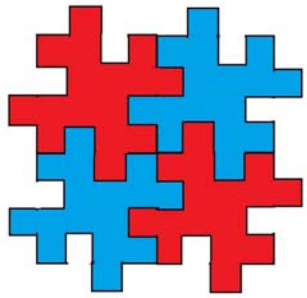
Note again the area remains at one square unit while the length of the bounding curve after an infinite number of iterations becomes infinity. Looking at the ratio of 14 to 6 for the generating element we also can conclude that-

$$L(n)=4(7/3)^n$$

There are an additional infinite number of generating elements which produce fractal bounding curves whose enclosed area always remains unity for any generation n considered. It is the integers in the Hausdorff dimension which determine the precise length of the bounding curve at iteration n.

You will notice that the inner area produced by the boundary curve at the nth iteration are like jig-saw puzzle pieces that nicely fit into each other making for a continuous flat surface without open spaces. That is, they would make perfect floor or wall tiles as shown-

MERGING THE FIRST GENERATION OF THE DOUBLE PULSE INTO A CONTINUOUS TILE PATTERN



A(1)=1 and L(1)=8

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