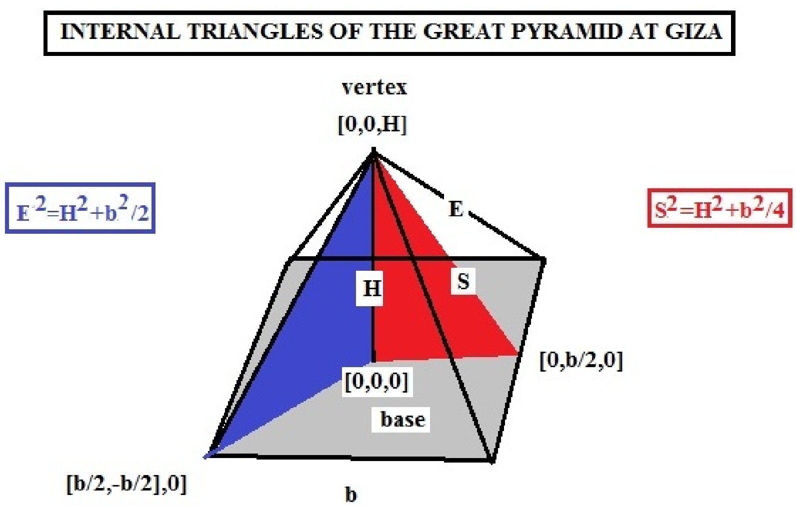
ANALYSIS OF THE DIMENSIONS OF THE GREAT PYRAMID AT GIZA

People have tried over last few hundred years to understand the construction methods and dimensions used by the ancient Egyptians in the building of the Great Pyramid at Giza just outside of present day Cairo . Associated with many of the existing studies concerning this pyramid, there has developed a great deal of pseudoscientific speculation concerning the presence of hidden numbers such as π and the golden ratio φ in its dimensions. Even wilder speculations involving western religion role(non-existent in 2560 BC), extra-terrestrial construction aide, and use of pyramid structures to sharpen knives and keep fruit fresh have appeared in the literature. We want here, as a compliment to our earlier article on pyramid construction(see http://www2.mae.ufl.edu/~uhk/PYRAMID-TECHNOLOGY.pdf), to concentrate on obtaining a better understanding of the pyramids dimensions and do so without bringing in any non-scientific nonsense as is often presented on the History Channel.

Our starting point is the following schematic of the of the Great Pyramid of Cheops(alias Khufu) on the Giza plateau. The schematic includes two vertical internal right triangles which may have been used by the architects to guide them in construction. Here is the figure-



One knows from direct measurements that the base area $A_{base}=b^2$ (shown in grey) is $(755.79)^2=571218.5241\ ft^2=13.1$ acres. The base is also perfectly aligned with a north-south axis. The pyramid has a present day height of 455ft although its actual original height was H=481ft. The difference in height is due to cladding removal about 1356 AD for the re-building of mosques in downtown Cairo. Using the original pyramid height H, one finds the acute base angle of the red triangle shown to be-

$$\theta_{red} = \arctan(2H/b) = \arctan(1.272840) = 57.6058\,\text{deg}$$

For the blue triangle the acute base angle is-

$$\theta_{blue} = \arctan(H\sqrt{2}/b) = \arctan(0.900034) = 46.6536\deg$$

Using the Pythagorean Theoerem we have the edge length to be-

$$E = \sqrt{H^2 + b^2/2} = 719.01 \text{ ft}$$

and the slant line length to be-

$$S = \sqrt{H^2 + b^2/4} = 611.69\, ft$$

As discussed in our earlier note, the stones used in the pyramid construction where probably dragged up to the construction level along this slant line using a wooden sled.

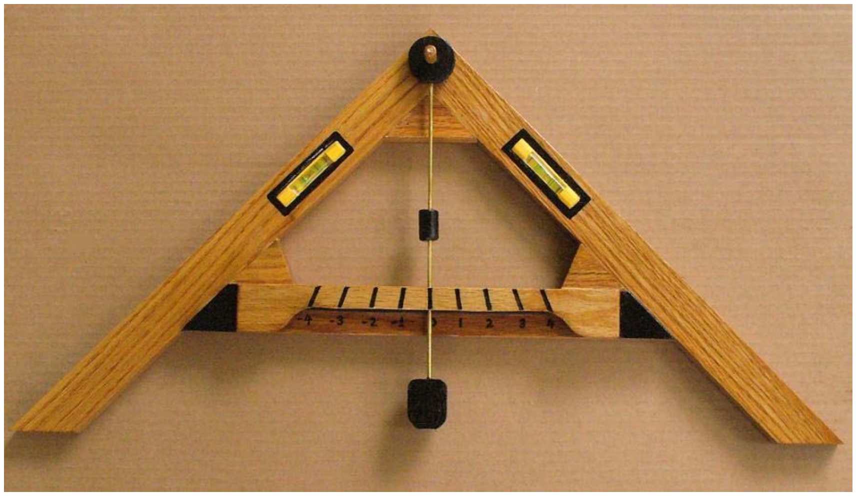
The volume of the Great Pyramid was originally-

$$V = \frac{1}{3}b^2 H = 91{,}585{,}370\, ft^3$$

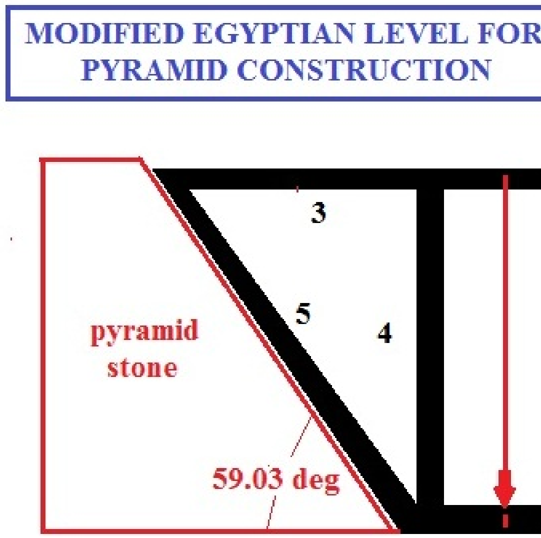
A close-up view of the Cheops Pyramid indicates the average stone size used was about 3x3x3=27 cubic ft. This would require the use of some 3.4 million stones of this size for building the entire pyramid. Assuming a ten hour work day and a total construction time of 20 years, this would require the placement of about one such stone per hour. This is not an unreasonable task for individuals working in the pre-machine age using only ropes, copper chisels, sleds with lubricated runners, levels and a means to measure distances (1 cubit=7 palms and 1 palm=4digits) and tangents of angles(sekeds), plus an almost unlimited supply of manual labor.

In the past people have tried to ascribe deeper mathematical meaning to the great Pyramid dimensions. Wildly speculative books in the 18 hundreds by British authors such as Smyth and Taylor, tried to fit these dimensions into mathematical expressions involving the Golden Ratio φ=(1+sqrt(5)}/2 and the irrational number π. To do so, they had to change the original statement by the Greek historian Herodotus(485-425BC) concerning the pyramid dimensions to something which fit their theory. After doing so they came up with such interesting, but clearly wrong statements, that H=(b/2)sqrt(φ) and V=sqrt(φ)b^3/6.The actual dimension statement by Herodotus was that the Cheops Pyramid has a square base of side-length b=800 Greek feet and a height H of this same amount. Clearly these numbers are much larger than the actual values of b and H.

Our own view is that there are no hidden mathematical constants in the Great pyramid, but that the dimensions nevertheless make sense when considering the ancient Eqyptian's grasp of geometry and the measuring instruments used by them. They measured the tangent of angles in terms of the seked using the Egyptian level. A modern version of such a level, which I constructed in my workshop, looks like this-



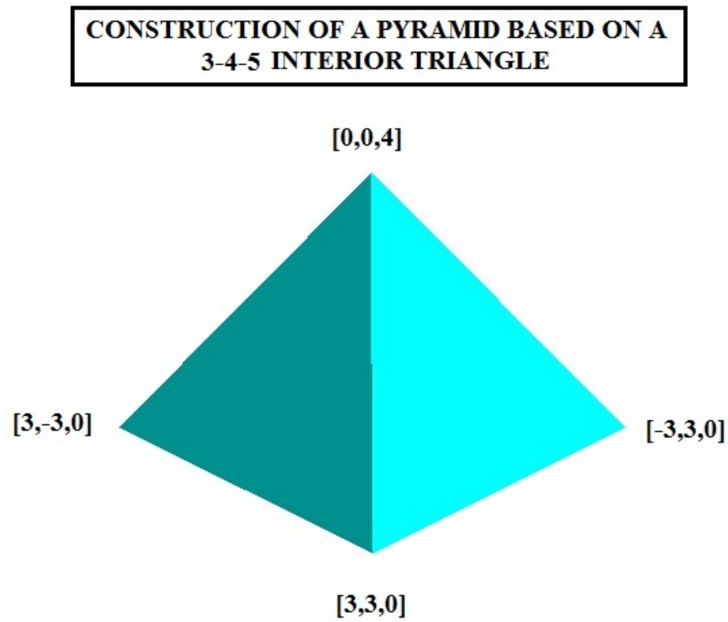
The level looks like an A frame with a plumb-bob free to pivot about a point at the apex of the frame. By placing the bottom of the A frame on a sloping surface one can read-off directly the level’s incline. Although the ancient Eqyptians of 2500BC where probably not yet aware of the Pythagorean Theorem, they must have noted when constructing their levels that a right angle will be formed at the pendulum pivot point when equal length bars are held together by a cross-bar equal to the square root of the two times the distance between their ends. Even if the concept of the irrational root of two was not yet clear to them, this combination always led to a right angle at the apex. Also someone involved with level construction must have noticed that a right angle can also be created by connecting three pieces of wood having lengths 3, 4, and 5 into a triangle. That is, they probably recognized the elementary Pythagorean integer triplet [3,4,5] some 2000 years before Pythagoras. If they indeed found the secret of the 3-4-5 triangle, it would then have been a simple matter to construct a modified Egyptian Level useful at all levels of the stone cutting and pyramid construction process. I would envision the angle measuring level to look as follows-



Although no such level has been found by Egyptologists either depicted in tomb hieroglyphics or in digs, I would not be surprised if such a find were made. It clearly is a tremendous time saving device in stone preparation . What makes one suspicious that the builders knew about 3-4-5 triangles is that the angle θ_{red} found above for the red triangle inside the generic figure is 57.6deg and thus differs from the angle θ of a 3-4-5 triangle by just 1.4 deg.

Notice that none of these observations require the need for introducing mathematical constants such as π or φ into the discussion. Instead it only relies on what measuring instruments were available at that time to the ancient Egyptians and the known degree of their mathematical knowledge (see Moscow Papyrus).

We finish up by constructing a 3-4-5 pyramid. In this case H=4, b=6, and S=5 and the five vertexes of the pyramid lie at [3,3,0], [3,-3,0], [-3,-3,0], [3,-3,0], and [0,0,4]. The resultant pyramid looks like this-



It has very much the appearance of the Great Pyramid of Cheops. Its four side surfaces form isosceles triangles of area-

$$A_{face} = \frac{b}{2}\sqrt{H^2 + (b^2/4)} = 15 \qquad each.$$

The pyramid volume equals-

$$V_{pyramid} = \frac{1}{3}b^2 H = 48$$

, and the surface area to volume equals 4x15/48=5/4 reciprocal length units.

What is important to remember here is that the Cheops's Great Pyramid as well as the present 3-4-5 pyramid (or other variations thereof) are constructed without needing to hide numbers such as π or φ in its dimensions. All that is really needed are the tools extant and manpower available at the time and sufficient zeal and funds to carry the building process to completion.

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