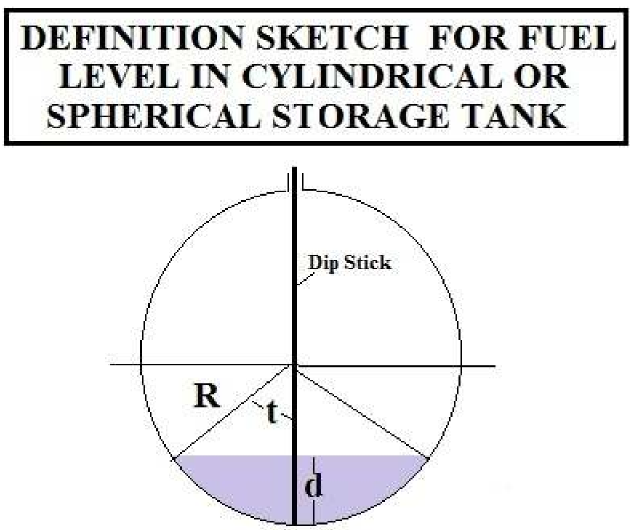
VOLUME LEVEL DETERMINATION IN FUEL TANKS USING A DIP-STICK

Many years ago I remember one of my undergraduate students, who was working at a local gas station to supplement his income, asking me how the depth level indicated on a dip-stick can predict the remaining volume of gasoline left in a cylindrical storage tank . The solution is not that obvious since the cross section of such storage tanks are not generally rectangular. More commonly they are either cylindrical (for gasoline) or spherical (liquid propane). Let us consider the latter two. Let the dip-stick measure of fluid depth in such tanks be d and the tank radius be R. The geometry for the analysis is as shown-



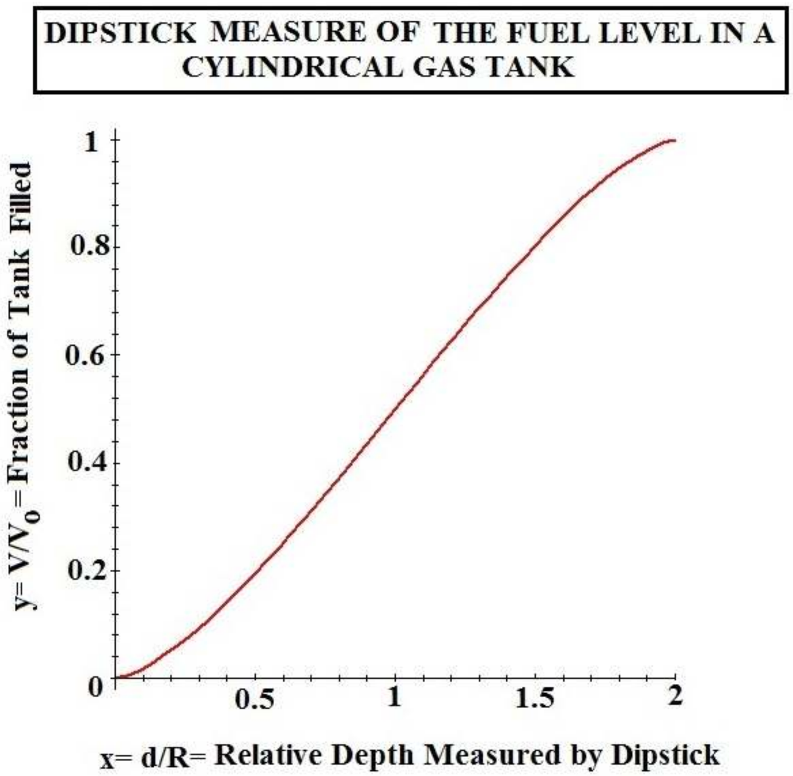
CYLINDRICAL TANK: Consider first a cylindrical cross-section tank of radius R and constant length L. The volume of liquid in such a tank when completely filled is simply $V_0=\pi R^2 L$. If the tank is only partially filled to depth d , the fluid volume, as seen from the above diagram, will be-

$$V_d=\pi R^2L[t/\pi]-(R-d)RL\sin(t)$$

Where t is the angle shown in the diagram. Thus the fraction of the tank filled with liquid will be-

$$y=V_d/V_0=(1/\pi)\{\arccos(1-x)-(1-x)\mathrm{sqrt}[x(2-x)]\}$$

where we have used the equality $\cos(t)=(R-d)/R=1-x$ with $x=d/R$. A plot of this function follows-



You will notice the non-linear behavior between dip-stick depth d and the volume of liquid remaining in the tank. Only a very rough approximation would allow one to say that y=0.5x. at points other than x=0, 1, or 2. Such an estimate becomes especially bad when the tank is nearly empty or nearly completely filled.

SPHERICAL TANK: Next we examine the spherical tank case using the same configuration shown earlier but expressing things in terms of spherical coordinates. Flipping the figure upside down, one has that the spherical bottom cone has a volume-

$$V_{sc}=\iiint r^2 sin(\theta)drd\theta d\varphi = 2\pi\int_{r=0}^{R} r^2 dr \int_{\theta=0}^{t} sin(\theta)d\theta = 2\pi x R^3/3$$

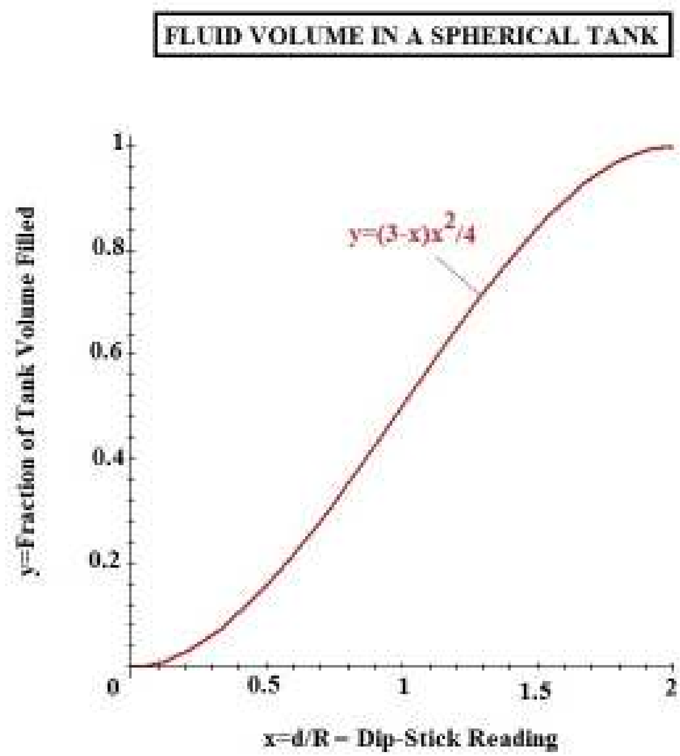
Here again x=d/R. Furthermore the volume V_f of the flat bottom cone, lying directly above the fluid level and having its vertex at the sphere center, is-

$$V_f=(\pi/3)(R^3(1-x)x(2-x)$$

Thus the volume of the fluid in the spherical tank expressed as a fraction of the total tank volume is-

$$y=3(V_{sc}-V_f)/(4\pi R^3)=(x^2/4)(3-x)$$

This simple cubic in x shows that y(0)=0, y(1)=0.5, and y(2)=1 as is to be expected. The complete graph over the entire range of interest looks as follows-



Note that its departure from a linear profile y=0.5x is larger than for the cylindrical tank. This comes from the fact that a shallow spherical cap of fluid with maximum radius r has a smaller volume than a cylindrical cap of the same depth and length L=r.

Finally let us look at a tank in the shape of an axisymmetric cone r=f(z). Here z is the vertical symmetry axis , r the radial coordinate, and the tank is assumed to extend from z=a to z=b where b>a. Here the volume of the filled tank is –

$$V_c=\pi\int_{z=a}^{z=b} f(z)^2\,dz$$

so that the volume fraction filled with liquid will be-

$$y = V_f/V_t = \frac{\int_{z=a}^{z=a+d} f(z)^2\, dz}{\int_{z=a}^{z=b} f(z)^2 dz}$$

For a simple cone where z=r and $0<z<H$, we have $y=(d/H)^3$. Thus if this cone is filled with fluid up to z=H/2 it contains only one eigths of the volume of the completely filled cone.