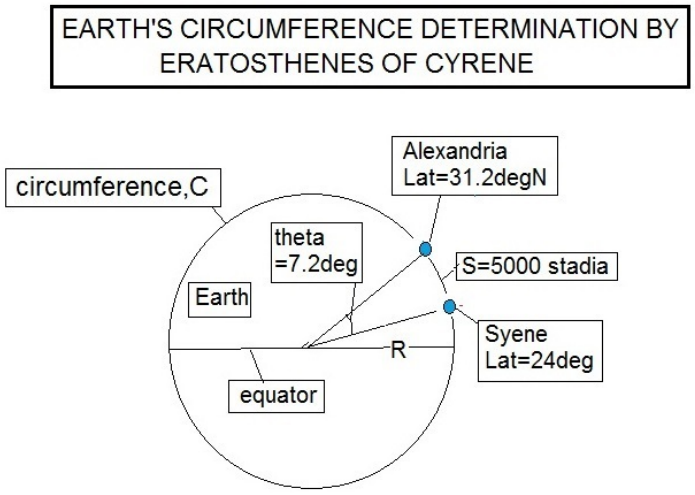
ERATOSTHENES AND THE EARTH'S CIRCUMFERENCE

The first relatively accurate measurement of the Earth's circumference C was made by the polymorph Eratosthenes of Cyrene(276-195BC) who was also chief librarian at the Library of Alexandria, invented the sieve method for finding prime numbers, and was the first to introduce a latitude-longitude grid for mapping the location of places in the then known world. We want in this brief article to show how he measured the Earth's circumference and suggest two alternate routes he could have taken for finding C by making use of the nearby Light House of Alexandria.

We begin with a schematic of his measuring approach for finding C. It looks like this-



C=S360/7.2=250,000 stadia=24,500 miles

He was located in Alexandria, Egypt at Lat=31.2N and knew that the town of Syene (now Aswan) was S=5000 stadia directly to the south at Lat=24N. It was also known that at the summer solstice the sun at noon at Syene was directly overhead because its image could be reflected at the bottom of deep wells. At the same date the sun at local noon in Alexandria was θ=7.2deg=0.12566 rad off of vertical. So from simple geometry he knew that-

$$C = 2\pi R = \frac{2\pi S}{\theta} = \frac{2\pi S}{(0.12566)} = 250{,}007\ stadia$$

It is usually assumed that one stade equals 516.75 ft=0.097869 miles. So, he deduced that the Earth's circumference should be-

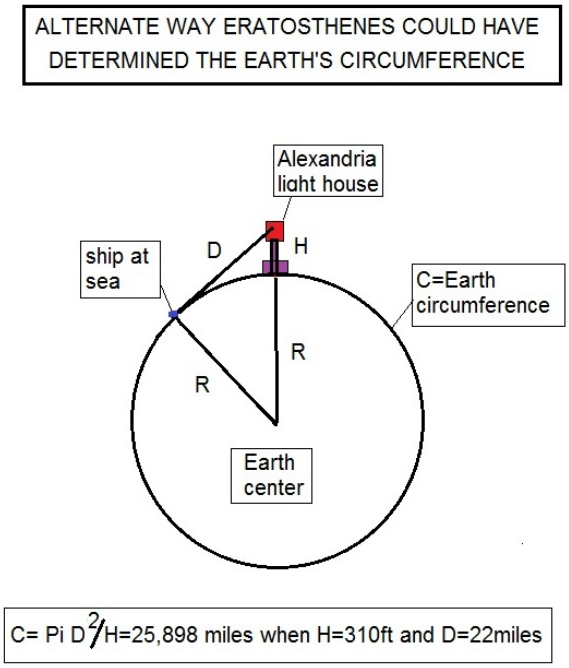
C= 24,468 miles

and its radius-

R=3894 miles

These results are remarkably close to the presently accepted value of C=24,875 miles and R=3959 miles. There is, however, some question as to which stadia measure Eratosthenes was using in his calculations. This stems from the fact that this length measure had its origin in the length of sports stadiums in the ancient world and these differed from country to country. It is said that Columbus used one of the shorter values of stadia to convince himself and the king of Spain that the distance to China going west from Europe was much shorter than it actually is.

I sometimes wonder why a sharp individual such as Eratosthenes did not realize that a much simpler way to find C would have been for him to walk over to the famous Light-House of Alexandria (built in 284 through 246 BC) and simply inquire what is the longest distance D a ship at sea at night could spot the signal fire near the top of the light-house. This information would have been known to navigators. I estimate that D was about 22 miles for a light house light signal at a height of H≈320 ft when viewed from sea-level.. Here is a schematic of the situation-



From the geometry we have a right triangle as shown. It says, via Pythagoras, that-

$$(R+H)^2 = D^2 + R^2$$

Since R>>H, we can write this as -

$D^2=2HR$

From it we have the Earth's circumference as-

$$C = \pi \frac{D^2}{H} = \frac{\pi\, 5280\, (22)^2}{320} = 25{,}088\, miles$$

A third way Erastosthenes could have found C would be to have noted the time difference of sunset at the bottom and top of the Alexandria light house looking west out over the Mediterranean. This time difference directly relates to the angle over which the Earth has rotated during the time interval. From this information one finds-

$$\theta = 2\pi\, t/(24 \cdot 60) \quad and \quad \theta^2 = 2H/(5280 \cdot R)$$

, when the time interval t is measured in minutes, the angle change θ in radians, and the light house height H in feet. Combining we have an Earth radius of-

$R=19.896\ (H/t^2)$ miles

Taking H=320 ft and t=1.25 minutes we get R=4075 miles and C=25,604 miles. Here we note that R goes as the reciprocal of the square of the time t. This makes it necessary to precisely know the time it would have take between sunset at sea level to that at the top in order to get close to the true circumference.

Of the three methods for finding C, Eratosthenes used only the first. The most accurate method would instead have been the Earth curvature measured by the furthest distance a ship could see the light signal at the top of the Alexandria Light House.

We point out that the Library of Alexandria, where Eratosthenes was employed, had its golden age during his presence in the second century BC. Such luminaries as Archimedes , Aristarchus , and Apollonius where his colleagues. The library (school) at Alexandria went into decline at the beginning of the Christian era including the accidental burning of many of its most priceless scrolls. The light house, which was one of the seven wonders of the ancient world, stood until a major earthquake brought it down in 956 AD.

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February 9, 2019
Gainesville, Florida