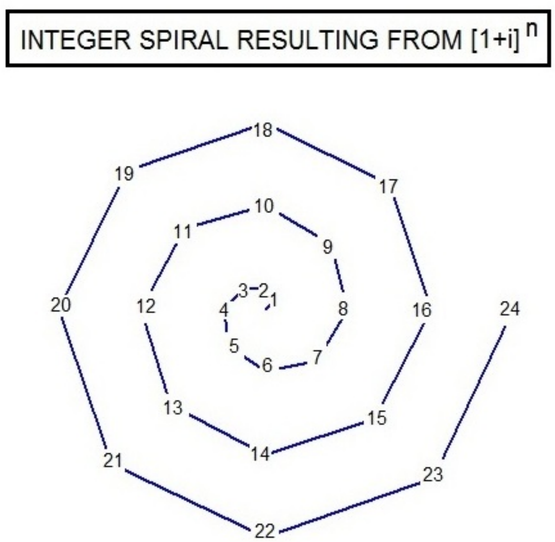
DEVELOPMENT OF THE HEXAGONAL INTEGER SPIRAL AND ITS ROLE IN PRIME NUMBER CLASSIFICATION

Several decades ago while teaching complex variables to a group of undergraduate engineering students here at the University of Florida we ran across the point function-

$$z[n] = (1+i)^n = 2^{n/2}\exp(i\pi n/4)$$

It was realized that when plotting these points in the complex plane that a spiral like pattern emerges. When these points are connected by straight lines one finds the following octagonal integer spiral-

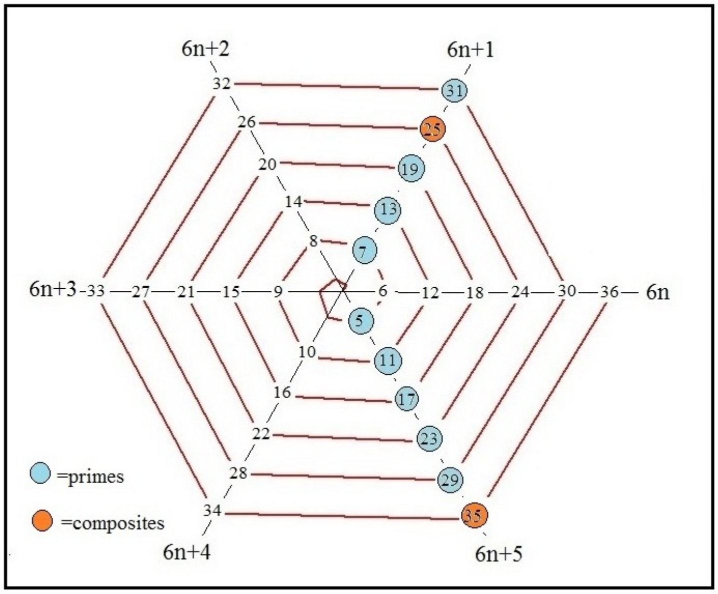


We note here that each turn of the spiral raises the integer by eight. I kept this result in mind some ten years later when I was studying prime numbers after my retirement. By means of a new point function f[N[={sigma(N)-N-1}/N we first realized at the time that all prime numbers beyond p=3 must have the form 6n±1 although some composites may also have this form. This fact triggered the thought that a convenient way to present primes graphically would be by means of a point spiral similar to the one studied earlier in our complex variable class except changing the magnitude from $2^{n/2}$ to |N| and the angle from nπ/4 to nπ/3. This will produce the following new representation for all positive integers N-

$$N=|N| \exp(i\pi n/3)$$

I call this the Hexagonal Integer Spiral. With its vertexes are connected one arrives at the pattern-

HEXAGONAL INTEGER SPIRAL



We have superimposed six radial lines on the pattern. These lines correspond to 6n, 6n+1, 6n+2, 6n+3m, 6n+4, and 6n+5 . The integers are located at the intersection of a radial line with one of the vertexes of the hexagonal spiral. What is most interesting about this result is that all primes greater than three have the form 6n+1 or 6n-1 and thus lie along just two of the radial lines 6n+1 or 6n-1(equivalent to 6n+5). They are shown as blue circles in the graph. It is amazing to me that this type of prime classification via hexagonal spirals has not been found before. Mathematicians have spent years trying to make sense out of the Ulam Spiral and have written elaborate programs concerning the quasi-random appearance of primes in their resultant patterns. A simple morphing could have saved them a lot of time. Note that in terms of modular arithmetic, primes of form 6n+1 have the property p mod(6)=1 and those where p=6n-1 are characterized by p mod(6)=5. So for instance one knows at once that the prime-

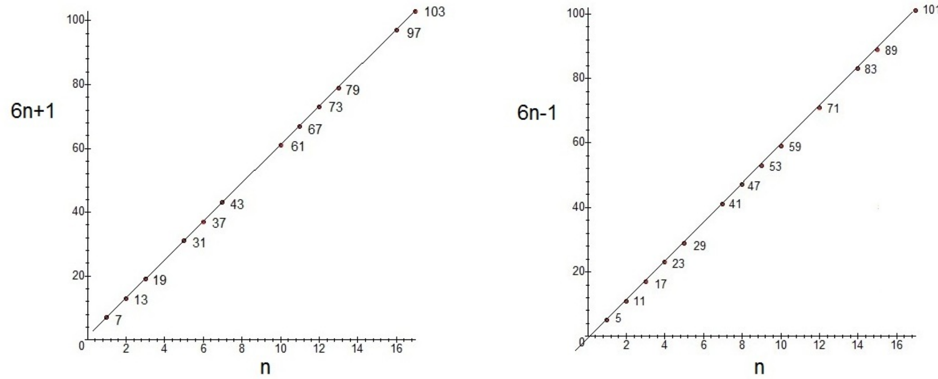
P=190898885521 has p mod(6)=1 and so lies along 6n+1

and prime-

p=427419669071 has p mod(6)=5 and so lies along 6n-1

We can expand the range of prime locations along the 6n±1 radial lines by looking at the first 25 primes 5 or greater. This yields the following patterns-

PRIMES ALONG RADIAL LINES 6n+1 AND 6n-1



primes along either radial line differ from each other by factors of six

gaps at 25,49,55,85,91,35,65,77,95 are composites

Note that the primes along either radial line differ from other primes along the same line by factors of six. So 97-43=54=6(9) and 101-23=78=6(13). The gaps along these lines correspond to compound numbers staring with semi-primes N=pq. So the gap noted between 71 and 83 corresponds to the semi-prime 77=7x11.

There are two types of primes which have been studied extensively in the literature. The first of these are the Mersenne Primes-

$M(p)=2^p-1$ which are prime only for certain primes p.

Some of the Mersenne Primes are 7, 31,127, and 8191. They all have M mod(6)=1 and hence lie along the radial line 6n+1.

The second set of primes are referred to as Fermat primes. They have the form-

F(n)={2^2^n}+1 and are indeed primes for n=1, 2, 3,and 4

However for n=5 or greater one finds only composits for all larger ns tried so far. There is of course a chance that one of the larger ns will produce a prime. We have –

F(1)=5, F(2)=17, F(3)=257, and F(4)=65537

They each have F mod(6)=5 and so lie along the 6n-1 radial line.

One could also define an even more general prime given by-

$$L[a,b,p]=a^{p}+b$$

where a and b are integers and p is a prime. Among the infinite number of possibilities one finds the primes-

L[3,-2,5]=241 with L mod (6) = 1

L[2,15,11]=2063 with L mod(6)= 5

L[4,-3,7]=16381 with L mod(6)= 1

L[5,6,13]=1220703131 with L mod(6)= 5

As with all primes greater than three, the L primes lie along either the 6n+1 or 6n-1 radial lines in a Hexagonal Integer plot. An even more general form, which contains both the Mersenne and Fermat primes as special cases, is-

$$T[a,b,c]=a^{c}+b$$

There are an infinite number of such T primes including T[2,3,6]=67 , T[3,2,4]=83, and T[18,6,5]=34012229. A really large T prime occurs for-

T[202,19,77]=

325128804688968025100617563248886662955586676797379056837536088010228557744866641530479897492245877092763324430071793820027711387608138952525774217259673520606526232937394025269 .

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