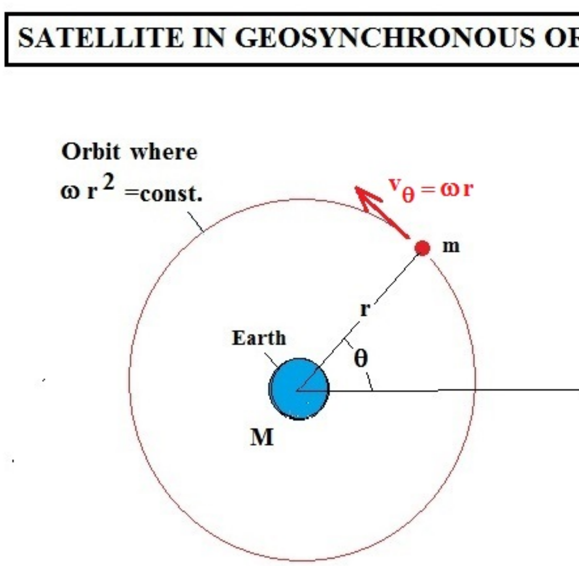
GEOSYNCHRONOUS EARTH SATELLITES

Perhaps the most important contribution of the world's space efforts over the last half century has been the advent of geosynchronous earth satellites. These devices have made global communications possible connecting every point on earth with any other point. Although the use of such satellites was mentioned as early as 1923 in Hermann Oberth's book on space travel, it was the science fiction writer Arthur C. Clark who popularized the idea of geo-satellites for radio communication in 1945. The first near earth satellite (Sputnik) was launched in 1957 and the first successful geosynchronous satellite (Syncom2) was placed into earth orbit in 1963. Since that time hundreds of these satellites now exist in nearly circular orbits some 35,800 km above the earth's equator.

It is our purpose here to discuss the basic mechanics behind a satellite in geosynchronous orbit moving at speed rω at height r above the earth's center. Here ω=dθ/dt matches the earth's rotation rate so that the satellite will appear fixed above a given point along the equator. A schematic of the set-up is as shown-



In this picture we are looking at the earth and orbit from a point above the north pole so that the satellite and the earth both rotate counterclockwise at the same angular speed ω. The basic equation governing the satellite motion is Newton's Second Law F=ma. In polar coordinates this equation has the two components-

$$m(\ddot{r} - r\dot{\theta}^2) = \frac{-GMm}{r^2} \quad and \quad m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0$$

The second equation here is equivalent to the conservation of angular momentum (and also equivalent to Kepler's Second Law of planetary motion). It reads-

$$r^2\omega = Const.$$

For a pure circular orbit, the first equation reduces to-

$$\omega^2 = \frac{GM}{r^3} = \frac{gR^2}{r^3}$$

since at the earth's surface we have $GmM/R^2=mg$.
From this last result we can conclude that-

$$\frac{H}{R} = -1 + \left[\frac{g}{\omega^2 R}\right]^{1/3}$$

equals the height H at which a geosynchronous satellite must orbit above the equator. Substituting in the values of $g=9.8066$ m/s^2, $R=6.371\text{x}10^6$ m, and $\omega=2\pi/(3600\text{x}24)=7.2722\text{x}10^{-5}$ r/s, we get-

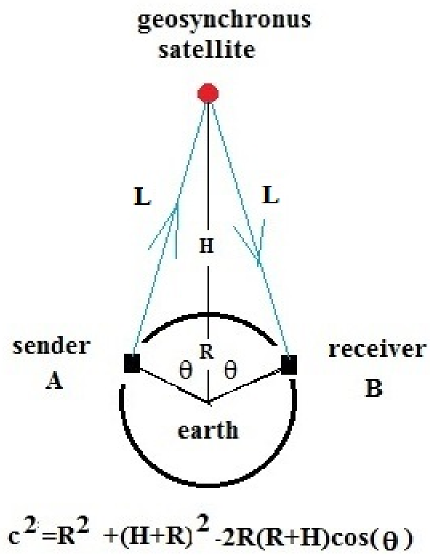
$$\frac{H}{R} = -1 + \sqrt[3]{\frac{9.8066 \cdot 10^4}{7.2722^2 \cdot 6.371}} = 5.627$$

Thus a geosynchronous satellite must be placed at a height of-

$$H = 6.371 \cdot 10^6 \cdot 5.627 = 35{,}849.6\, km = 22{,}275.9 \textit{ miles above the equator}$$

This number is close to the value of H=35,786 km found in handbooks.

You may have wondered why home TV satellite receivers always point approximately south at a fixed angle with respect to the horizon. This is because one is receiving the reflected signals from a particular geosynchronous satellite sitting above the equator (probably above Ecuador for east coast viewers). The angle above the horizon to which a parabolic receiver must be pointed is approximately equal to $\pi/2$-LAT, where LAT is the local latitude. Use of several of such satellites placed at equal distances around the equator makes possible almost instantaneous communication between any two places on earth (arctic regions excluded). A slight delay in such communications when they are two-way is noted because of the finite speed of light at $c=3\cdot10^8$ m/s. For instance in a two-way conversation with live pictures between Tahrir Square in Cairo and a home news office in New York, one can expect a delay of about 4L/c where c is the speed of light and L the distance from the sender or receiver to a geosynchronus satellite as shown in the figure-



By the law of cosines we have $L^2=(H+R)^2+R^2-2R(H+R)\cos(\theta)$, so that the delay will equal-

$$\tau = \frac{4}{c}\sqrt{(H+R)^2 + R^2 - 2R(H+R)\cos(\theta)}$$

Now the great circle distance from Cairo to New York is about 9 million meters so that $\theta=9\mathrm{x}10^6/(2R)$. Using $R=6.37 \times 10^6$ m, $H=35.8 \times 10^6$ m, and $c=3 \times 10^8$ m/s, we have a time delay of $\tau=0.501$ seconds. This approximate half second delay is noticeable when observing the reaction time of the reporters. However the picture quality is amazing. No one could have dreamed of being able to do this several decades ago.

One can also use the above equations to find the orbit period $\tau=2\pi/\omega$ of any satellite of mass m in a circular orbit about a much larger mass M. We have-

$$\tau^2 = \frac{(2\pi)^2}{GM} r^3$$

From this we note that the square of the period is proportional to the cube of the distance to the center of the large mass M and also inversely proportional to M. This is essentially Kepler's Third Law. We can use this result to calculate the mass of the earth M by measuring the period of any earth satellite in a circular orbit since the value of the universal gravitational constant $G = 6.6738\cdot10^{-11}\ m^3kg^{-1}s^{-2}$ has been known since the time of Henry Cavendish(1731-1810). Sputnik was the first near earth satellite and its orbit period was about 96 minutes with a mean altitude of H=577 km. I remember getting up quite early one morning back in October of 1957 when it was still dark to watch this satellite streak overhead. A very impressive sight. Let us see how we can use this data to find the earth's mass. We have from the above formula that-

$$M_{earth}=4\pi^2R^3/G\tau^2=4\pi^2(6.371+0.577)^3\cdot 10^{18}/[6.6738\cdot 10^{-11}\cdot(96\cdot 60)^2]=5.98\cdot 10^{24}\text{ kg}$$

This number is very close to the official value of 5.97 x 10^{24} kg. The small discrepancy is undoubtedly due to the average height H used when the actual height range for Sputnik in its elliptical orbit was 215km<H<939km.

In a similar manner one can estimate the sun's mass by letting r be the mean distance from the sun to earth center of 1AU=149,597,871 km and take the 1 year orbit time as τ=365.2421*24*3600=31,556,926 s. Solving, we find the sun-earth mass ratio to be-

$$\frac{M_{sun}}{M_{earth}}=(\frac{96\cdot 60}{31.55688\cdot 10^6})^2(\frac{149.597\cdot 10^9}{(6.371+0.577)\cdot 10^6})^3=3.325\cdot 10^5$$

The best estimates for the sun mass is 1.988 x 10^{30} kg compared to the earth's mass of 5.97 x 10^{24} kg. The ratio is 3.329 x 10^5, again in good agreement with the above.

The period of a satellite orbiting the moon in a circular orbit can be quickly calculated using Kepler's Third Law that τ^2 is proportional to r^3/M. The moon has a mass of M_{moon}= 7.342 x 10^{22} kg and a radius of r_{moon}= 1738 km . A geosynchronous satellite about the earth is located at r= R+H as already shown. So a satellite orbiting the moon at a few meters above the moon's surface will have a period given by –

$$(\frac{\tau}{24\cdot 3600})^2=(\frac{1.738}{6.371+35.849})^3(\frac{5.97}{0.07342})$$

yielding the value-

$$\tau=6507s=1.807\ hours$$

Most existing satellites which have orbited the moon have had highly elliptic orbits of large eccentricity and hence have orbit times differing considerably from this value.

Finally, you will note that if one wanted to know the time τ_{Ast} it takes to orbit a small asteroid of unknown mass M_{Ast} and unknown mean radius R_{Ast} , one could argue that the mass is proportional to the cube of the radius if composed of material similar to that of the earth. This would allow us to state that, for a circular orbit near the asteroid surface, the orbit period should be-

$$\tau_{Ast}=24(\frac{R}{R+H)})^{3/2}=24(\frac{6.371}{6.371+35.849})^{3/2}=1.41\ hours$$

This is quite an interesting result since it requires no knowledge of the asteroid's mass or radius other that the material should have the same composition as that of the earth and that the universal gravitational constant G remains the same everywhere in the universe.

One could also reverse things and use the measured orbital period about an asteroid to find its density relative to the mean earth value of $5.52 \cdot 10^3$ kg/m^3.