GRAPHICAL GENERATION OF PRIMES

It is known that any composite number can be expressed as the product of primes. Thus-

2379 =3 x 13 x 61   and   345927= 3 x 115309

Also one knows that a prime P times an odd number 2n+1 is always a composite odd number. This fact is supported by the following table-

	3	5	7	9	11	13	15	17
3	9	15	21	27	33	39	45	51
5	15	25	35	45	55	65	75	85
7	21	35	49	63	77	91	105	119
11	33	55	77	99	121	143	165	187
13	39	65	91	117	143	169	195	221
17	51	85	119	153	187	221	255	389

where the composite odd numbers in black are the product of the prime in the first column and an odd number in the first row.

If one next writes down the sequences of numbers starting from the second row through the seventh, we have-

3-9-15-21-27-33-39-45-51-57-63-69-75-81-87-93…

5-15-25-35-45-55-65-75-85-95-…

7-21-35-49-63-77-91- …

11-33-55-77-99-…

13-39-65-91-…

17-51-85-…

where the first member of each sequence is a low value prime number P given in order and the subsequent terms are the composite numbers-

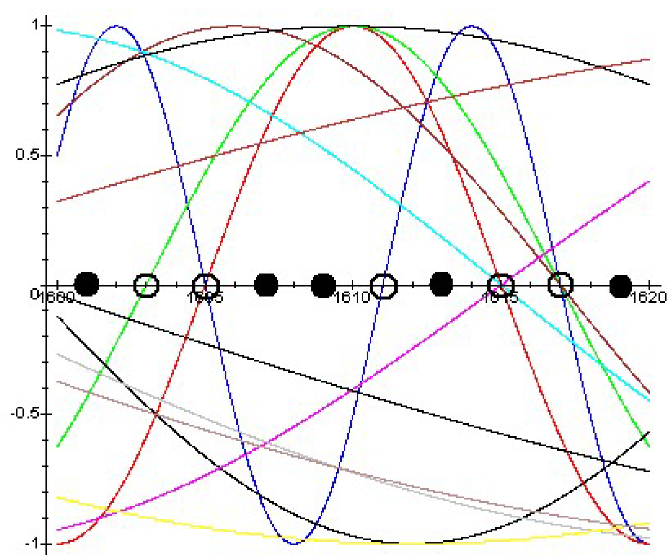
$$N = P(3+2k) \quad with \quad k = 0, 1, 2, 3, \ldots$$

The numbers missing from these sequences are the higher value prime numbers P. One can find these primes P graphically by noting that they must be located along the x axis where the sine functions-

$$F(x,P) = Heaviside(x-3P)\sin\left[\frac{(x-3P)\pi}{2P}\right]\ for\ P = 3, 5, 7, 11, .. \sim \sqrt{x}.,$$

do not vanish for the odd number x=2n+1. To demonstrate, let us find the values of x=N in the range 1600<n<1620 for which N will be a prime P. Plotting the functions F(x,P) for P=3,5,7,…41 we find the following-

GRAPHICAL DETERMINATION OF PRIMES IN THE RANGE 1600<N<1620 USING SINE CURVES CORRESPONDING TO P=3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, AND 41.



● =PRIME NUMBERS 1601, 1607, 1609 , 1613, 1619

○ =COMPOSITE NUMBERS

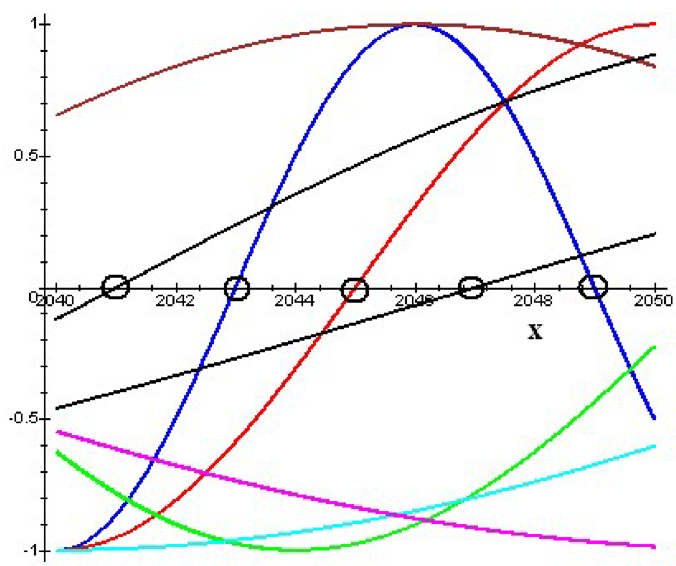
Clearly there are five odd number values at which the sine curves cross the x axis. These represent the location of composite numbers and are indicated as unfilled circles. The solid black circles are located where none of the sine curves cross and hence yield the prime numbers P=1601, 1607, 1609, 1613, and 1619. It took a total of

12 different sine curves to come up with this graphical result. To test a number N for primeness one needs to plot a number of sine curves equal to approximately-

$$\textit{Number of Sine Curves} \approx \frac{\sqrt{N}}{\ln(\sqrt{N})} = \frac{2\sqrt{N}}{\ln(N)}$$

Which, according to the prime number theorem, represents the approximate number of primes in 0<x<sqrt(N). If the number should turn out to be composite then this number of curves can be considerably less. If we take the example of the Merseene number N=2^{11} -1= 2047 we possibly could require 12 sine curves, however, one finds the need for only the first 8. Here is the graphical solution using F(x,P) with P=3,5,7,11,13,17,19,23 in the range 2040<x<2050.

GRAPHICAL DEMONSTRATION THAT THE MERSEENE NUMBER N=2^11-1=2047 IS COMPOSITE



Curves crossing the x axis indicates that numbers

N=2041, 2043, 2045, 2047, and 2049 are all composite.

It is seen that the sine curve corresponding to P=23 crosses the axis at x=2047 and hence this Merseene number is composite and equal to the product 23 x 89. Note that both 23 and 89 are primes. Also one finds the additional benefit of the graphics in that it shows the other four odd numbers in the given range of x to also be composite. The drawback of this method for finding primes by the elimination of composite numbers is similar to that encountered with the Sieve of Eratosthenes

approach. Namely, when the number N gets large, the number of calculations required will become large. For instance, to test a hundred digit number x=N would require the use of some

$$\frac{2\cdot 10^{50}}{\ln[10^{100}]}\approx 8.68\cdot\ 10^{48}$$

sine curves for a graphical solution or about the same number of divisions N/P with P=3,5,7,…. Public key cryptography is based upon the fact that it is very difficult to factor such large numbers N into the product of two primes using even the fastest electronic computers available.

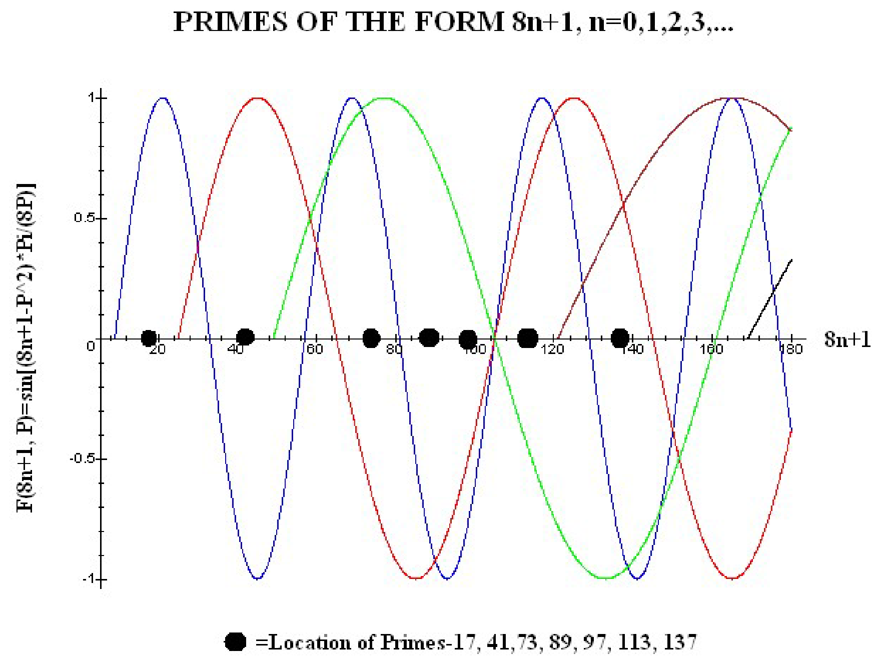
As a final consideration, consider finding the nth prime generated by the subclass of odd numbers-

x=8n+1=9, 17, 25, 33, 41, 49, 57, 65, 73, 81, 89, 97, 105, 113, 121,129, 137,…

In view of previous discussions, these all lie along the diagonal line y=x>0 where it intersects the Archimedes spiral r=(4/π)θ and are separated from each other by multiples of 8. Regrouping shows that all points corresponding to zeros of the sine curve-

$$F(x,P)=\sin\left[\frac{(x-P^2)\pi}{8P}\right]\ for\ P=3,5,7,11,13,...$$

will be composite numbers. Those numbers remaining will be prime numbers. We demonstrate for values of x=8n+1 in the range 1<x<180.



Note that the number of sine curves required for finding all inclusive primes through 8n+1=180 is just five. All prime numbers of the type 8n+1 up to 1049 are given in the following table-

n	8n+1	n	8n+1	n	8n+1	n	8n+1
2	17	32	257	65	521	101	809
5	41	35	281	71	569	107	857
9	73	39	313	72	577	110	881
11	89	42	337	74	593	116	929
12	97	44	353	75	601	117	937
14	113	50	401	77	617	119	953
17	137	51	409	80	641	122	977
24	193	54	433	84	673	126	1009
28	233	56	449	95	761	129	1033
30	241	57	457	96	769	131	1049

If one is interested in only checking a single number N for primeness, it will be simpler to simply look at the quotients N/P, P=3,5,7,11,.. For example, the Fermat number $N=2^{32}+1$, which is of the form 8n+1, has the prime divisor P=641 and hence is composite-

$$2^{32}+1=4{,}294{,}967{,}297=641 \cdot 6{,}700{,}417$$

and lies at the intersection of the diagonal line $y=x$ and the 641^{st} turn of the Archimedes spiral $r = (4/\pi)\theta$.

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