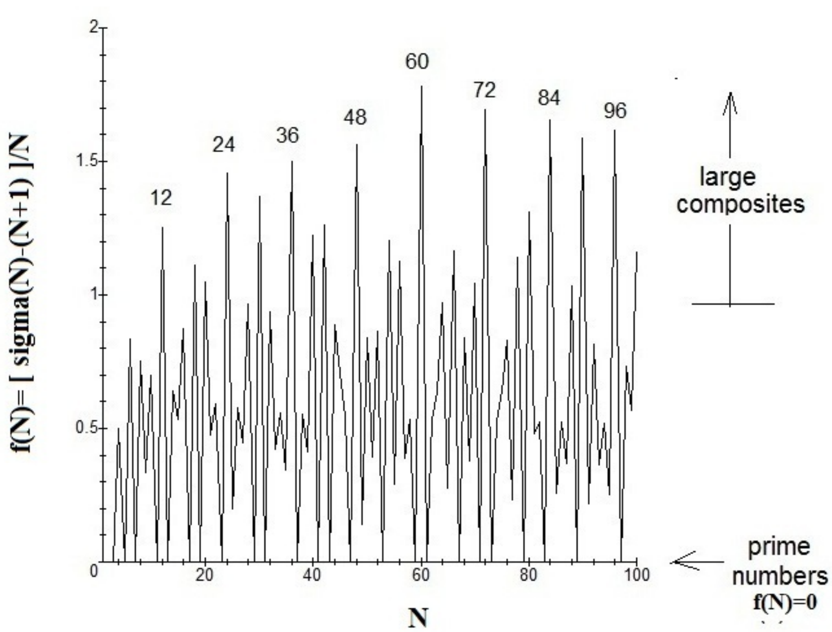
PROPERTIES OF THE HEXAGONAL SPIRAL AND
THE LOCATION OF PRIMES

Several years ago while studying the properties of integers we came up with a new number quotient f(N) which clearly distinguishes between prime and composite numbers. We have termed this point function the number fraction and defined it as-

$$f(N) = \frac{[\sigma(N) - N - 1}{N}$$

Here the function σ(N) is the sigma function of number theory. It represents the sum of all divisors of a number N. What is interesting about f(N) is that it is a rational point function unique for any positive integer. It vanishes only if N is a prime. Values of f(N) greater than zero are composite numbers with those greatrer than about 1.5 can be considered as super-composites. A plot of f(N) for the first hundred integers follows-

FIRST HUNDRED NUMBER FRACTIONS, f(N)



If you count the number of primes lying between N=5 and N=100 one finds precisely twenty three primes 5-7-11-13-17-19-23-29-31-37-41-43-47-53-59-61-67-71-73-79-83-89-97. You will note that these primes may be broken up into just two groups-

6n-1=5,11,17,23,29,41,47,53,59,71,83,89

and-

6n+1=7,13,19,31,37,43,61,67,73,79,97

The distance between primes in each of these groups is always a factor of six. However some of the 6n±1 numbers, such as 35,65,77,25,4955,85, are composite. Checking the values of N for which f(N)=0 out to at least N=10,000 yields the form 6n±1 without exception. This fact allowed us to conclude that –

A necessary but not sufficient condition is that any prime five or greater, has the form 6n±1. In modular arithmetic notation this means primes within the allowed range must have p mod(6)=1 or p mod(6)=5 without exception.

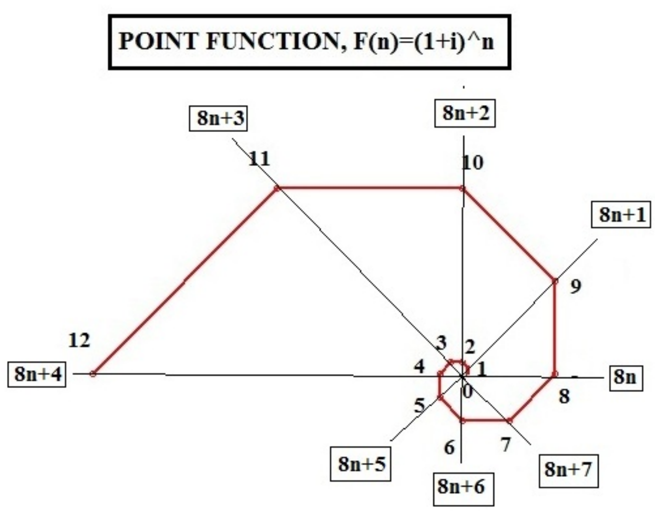
Just to show that this observation continuous to hold for very large integers consider the 34 digit long prime-

p= 3678194573147876291730796721347409

It can be factored into p=6(613032428857979381955132786891235)-1 or the equivalent p mod(6)=5. The three closest neighboring primes to p are found at p=+242 ,p=+158; and p=-292. all three are of the form 6n+1. The reason that the first two primes 2 and 3 are omitted from the discussion is that they are the only known primes which do not fall under the 6n±1 category.

When dealing with semi-primes N=pq the value of f(N) will be close to zero but not zero. However, since p and q are both primes, a semi-prime N=pq must also be of the form 6k±1 and so have N mod(6) be +1 or 5.

Our purpose in this note is to use the above information to present a geometrical interpretation of prime and composite numbers. We came upon such a representation by going back to our complex variable class which I taught some fifteen years ago. We noticed in the class that $F(z)=(1+i)^n$ produced an interesting figure in the z=x+iy plane in the form of a spiral with some radial lines separated by $\pi/4$ radians as shown-

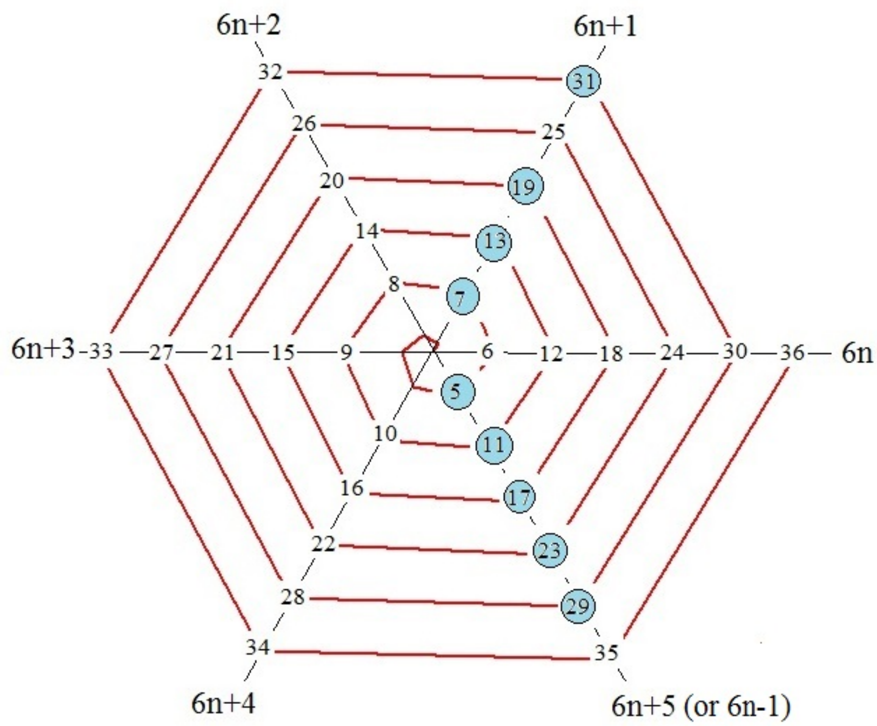


On recalling this curve and also being aware of the Ulam Spiral, and the 6n±1 condition for primes, I came up with a new hexagonal spiral whose vertexes represent the location of all integers. The spiral is represented in polar coordinates by the points-

$$[r,\theta]=[N,\frac{N\pi}{3}] \quad N=1,2,3,4,...,$$

If we next draw six radial lines ending at the origin and passing though the spiral vertexes one arrives at the following picture-

HEXAGONAL INTEGER SPIRAL AND THE LOCATION OF THE FIRST FEW PRIMES FIVE OR GREATER



We have completed the spiral by connecting neighboring vertexes by straight red lines. Note the six radial lines allow us to beautifully separate prime from composite numbers. It says, for example, that all odd numbers of the form 6n+3 must be composite as must the numbers corresponding to all even integers. The primes lie strictly along the 6n±l lines and pass through the spiral vertexes. This type of integer representation is far superior to the standard Ulam pattern where primes lie along multiple scattered short diagonal lines. It was not until about a decade ago that we showed that the Ulam spiral says little more than the fact that primes above p=3 are odd numbers. Despite of this fact today many mathematician-programmers still wrongly believe that the Ulam pattern reveals additional secrets about prime numbers. For a more detailed discussion of why they are wronh can be found by typing into the google search engine the words morphing ulam kurzweg.

I sometimes refer to the above diagram as a spider web since it resembles it in several ways. Indeed, a spider weaves its web by first laying some radial lines and then connecting then by a continuous spiral-like sticky strand. A picture of a typical spider web follows-

SPIDERWEB AS SEEN IN NATURE

(source-photography. nationalgeographics.com)

The gaps in the primes lying along the radial lines 6n±1 correspond to semi-primes N=pq and higher such as triple primes N=pqr. For example, the semi-prime N=17x29=493 has the form 6(82)+1. It fills the gap along the 6n+1 radial line between the primes 487 and 499.The value of f(493)=46/493 lies near zero (but not zero) as will be the case for most semi-primes N composed of the product of two primes p and q.

In general it will take an additional test to check whether a number 6n±1 is prime or not. The simplest of such tests is simply to divide the number N=6n±1 by all primes less than sqrt(N). If any of these divides N without leaving a remainder, then N will be a composite. For example, N=91=6(15)+1 and sqrt(91)=9.534. So we try division by 3,7,9 and see that p=7 goes exactly 13 times into N so that N=91 is a composite. There are also more extensive prime tests build into most computer math programs which answer at once whether 6n±1 is composite or prime.

Another interesting property of the above hexagonal spiral-radial line pattern is the location of double primes. Double primes are defined as having the form p=6n+1 and q=6n-1. That is they differ from each other by 2 units and lie symmetrically about the 6n radial line. Looking at the above diagram we have the double primes [[5-7] [11-13],[17-19], and [29-31]. Note that [23-25] is not a double prime since 25 is a composite.

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