IMPACT SPEED OF BULLETS FIRED STRAIGHT UP INTO THE AIR

Videos of Libyans freedom fighters in recent months have often shown them firing their rifles and machine guns straight up into the air at random during celebrations of local victories. It is clear that many of these young men, often in their teens, are unaware of the potential lethality of such acts. We want to here to carry out a rough calculation to determine if the impact velocity of their spent bullets returning to earth will pose a danger to people located nearby.

Let us first look at the approximate height reached by one of these bullets. From Newton's Second Law we have-

$$mV\frac{dV}{dx}=-mg-\frac{1}{2}C_D\rho AV^2$$

Here x is the vertical distance from the launch point, m is the mass of the bullet,V is the time-dependent bullet speed, ρ the air density, g the acceleration of gravity, A the bullet cross-section, and C_D the drag coefficient. The drag coefficient varies with speed but on average can be taken as about C_D=0.4. Letting $k=\rho C_D A/(2m)$, we can solve this equation to find the maximum height to which the bullet travels. It is-

$$H=\int_{x=0}^{H}dx=\int_{0}^{V_0}\frac{VdV}{g+kV^2}=\frac{1}{2k}\ln[1+\frac{kV_0^{\ 2}}{g}]$$

Here V_0 is the muzzle velocity. If we now consider the standard AK-47 Kalashnikov assault rifle, we have Vo=715m/s, bullet mass m=$7.8\text{x}10^{-3}$ kg, and cross section= $A=(\pi/4)(7.62\text{x}10^{-3})^2=4.56\text{x}10^{-5}\text{m}^2$. With $g=9.81\text{m/s}^2$ and air density of $\rho=1.29\ \text{kg/m}^3$, the value of k becomes-

$$k=\frac{\rho C_D A}{2m}=(\frac{1.29\cdot 0.4\cdot 4.56}{2\cdot 7.8})10^{-2}=1.51x10^{-3}\ in\ MKS\ units$$

Thus, the height the bullet reaches(assuming no tumbling) will be –

$$H=331.12\ln(78.69)=1445.51\,meters$$

If one next reverses the calculations and assumes a bullet is dropped with zero initial speed from this height, one has the equation-

$$V\frac{dV}{dx} = g - kV^2 \quad subject \quad to \quad V(0) = 0$$

Note that this time x measures distance downward and one of the signs on the right side of the equation is changed. Solving this equation, we find the impact speed of the spent bullet to be-

$$V_i = \sqrt{(\frac{g}{k})[1 - \exp(-2kH)]} = 80.1\, meters/\sec$$

The actual impact speed will be even smaller than this since the returning bullet is likely to be tumbling and thus producing a larger drag force.

This type of speed of a little over 260ft/sec is unlikely to penetrate anyone's head. Thus , although random shooting into the air is foollish and wastes valuable ammunition, it is not life threatening and the chance of even getting hit by a returning bullet is miniscule despite of the hundreds of rounds being fired into the air.

In connection with this analysis, it is also worth while to comment on one of the urban legends extant in this country. The legend claims that dropping a penny off of the Empire State Building can kill someone walking on the sidewalk below. This is nonsense since , even in the absence of air resistance, the impact speed will be only-

$$V_i = \sqrt{2gH} \approx 85.5\, m/s$$

when dropping the penny from the 102 floor observation platform. This might sting a bit or possibly damage an eye, but it will not kill. Time for the TV program Myth Busters to get busy on this problem.