# HIGH ALTITUDE PARACHUTE JUMPS

We want here to examine the speed as a function of height a parachutist will experience during a jump from heights above 100,000ft. The model describing this process is one where the individual will accelerate downward due to gravity while being slowed down by an altitude varying drag force. In mathematical terms one has-

$$v(dv/dy)=-g+[(c_D/2)\rho v^2 A/m] \quad \text{subject to} \quad v(H)=0$$

where v=speed, y=height, $g=9.8m/s^2$ the acceleration of gravity, m= parachutist's mass, A= his cross-sectional area, ρ=gas density, and $c_D$ the drag coefficient. In this equation both the drag coefficient and the density vary with height making the equation highly non-linear. For our calculations we will need the pressure, temperature, density and sound speed for the standard earth atmosphere. These are given in the following table-

| Height y (km) | Presure(kPa) | Temp.(K) | Density($kg/m^3$) | Sound Speed(m/s) |
|---|---|---|---|---|
| 0 | 101.3 | 288 | 1.225 | 341 |
| 10 | 26.50 | 223 | 0.4135 | 299 |
| 20 | 5.529 | 217 | 0.0889 | 295 |
| 30 | 1.197 | 226 | 0.01841 | 302 |
| 40 | 0.2871 | 250 | 0.003996 | 317 |

We note that the sound speed goes as the square root of the temperature and is thus relatively little changed over the range of ground level to 100,000ft . c=300m/s is a reasonable approximation in this range. We can use the data from this table to obtain the approximate continuous pressure and density variations in exponential form. They read-

$$p=101.3\exp(-0.143y) \quad \text{and} \quad \rho=1.225\exp(-0.131y)$$

with y expressed in kilometers, p in kilopascals, and ρ in kilograms per cubic meter. The drag coefficient $c_D$ is dependent on Reynolds number and hence speed. Its typical range is 0.2 for a well streamlined body to about 1.5 for flat surface normal to the flow direction. In our calculation we will assume the jumper will try to streamline things and so have an average drag coefficient of $c_D$=0.5. We assume the jumper and his pressure suit will have a mass of 130kg and an exposed cross-sectional area of $A=1m^2$ . With these approximations the governing equation becomes-

$$v(dv/dy)=-g+\{[1.225v^2 \exp(-0.131y/1000)/[2(130)]\} \text{ with } v(H)=0$$

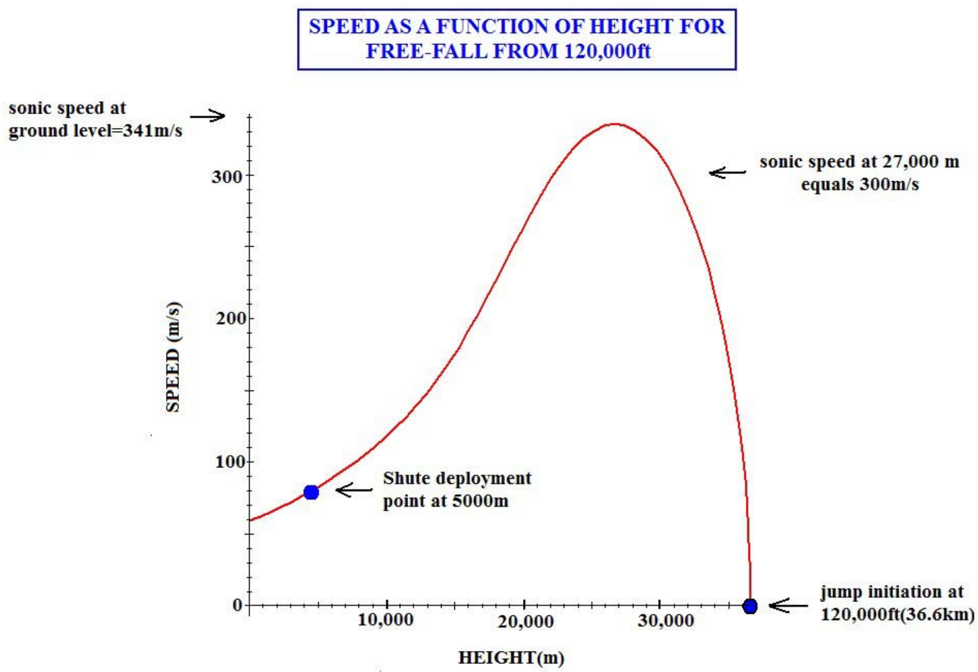
In this equation all terms are expressed in MKS units so that y is in meters and v in meters per second. Next letting $U=v^2$ , the equation reduces to-

$$dU/dy=-2g+\{a \exp(- b\, y)\}U \quad \text{with } U(H)=0$$

Here a=0.005772 and b=0.000131. We can solve this equation in closed form getting-

$$v=\sqrt{(g/b)\, e^{-(a/b)\exp(-by)}[Ei(1,-(a/b)\exp(-by))-Ei(1,-(a/b)\exp(-bH))]}$$

where Ei(n,x) is the exponential integral int(exp(-xt)/$t^n$, t=0..infinity). A plot of this function follows directly below-




The result is very close to the true downward speed a free-fall parachutist will encounter. **First there is an almost constant downward acceleration of 1g reaching a speed in excess of Mach one at about 27,000 meters. After that the atmospheric drag takes over and the jumpers speed will decrease continually due to atmospheric braking until reaching a speed of about 80 meters per second at y=5000m. At this point the parachute will deploy and a rapid deceleration will follow reducing the speed to about 5m/s for a safe ground landing.** The largest g forces will be felt during the parachute opening where one decelerates from 80m/s to 5 m/s in as little as 20m yielding a deceleration as high as 16g . To lower such high decelerations, multiple smaller chutes should be used and released sequentially.

Overall the pending free-fall jump should be safe with the largest dangers occurring during parachute deployment and during the supersonic portion of the descent. The supersonic speed experienced by the parachutist should not cause problems provided the spacesuit worn by the jumper is sufficiently sturdy to avoid ripping from the high shear forces encountered and the parachutist can maintain a steady orientation. The parachute deployment should be automated in case the parachutist blacks out.