# LARGE AND SMALL NUMBERS

When describing large or small units of length or time one uses a power of ten designation which involves Geek and Roman prefixes. Most will be familiar with terms such as kilogram($10^3$g) or microsecond($10^{-6}$s) but less so with powers of 10 $^{\pm 3n}$ when n≥3. Let us briefly give the accepted names and symbols for values ranging from $10^{24}$ to $10^{-24}$ and then point out some typical lengths and times occurring in this range. Here is the list-

Large Numbers | Small Numbers

| | | | |
|---|---|---|---|
| kilo(k) | $10^3$ | milli(m) | $10^{-3}$ |
| mega(M) | $10^6$ | micro($\mu$ ) | $10^{-6}$ |
| giga(G) | $10^9$ | nano(n) | $10^{-9}$ |
| tera(T) | $10^{12}$ | pico(p) | $10^{-12}$ |
| peta(P) | $10^{15}$ | femto(f) | $10^{-15}$ |
| exa(E) | $10^{18}$ | atto(a) | $10^{-18}$ |
| zetta(Z) | $10^{21}$ | zepto(z) | $10^{-21}$ |
| yotta(Y) | $10^{24}$ | yocto(y) | $10^{-24}$ |

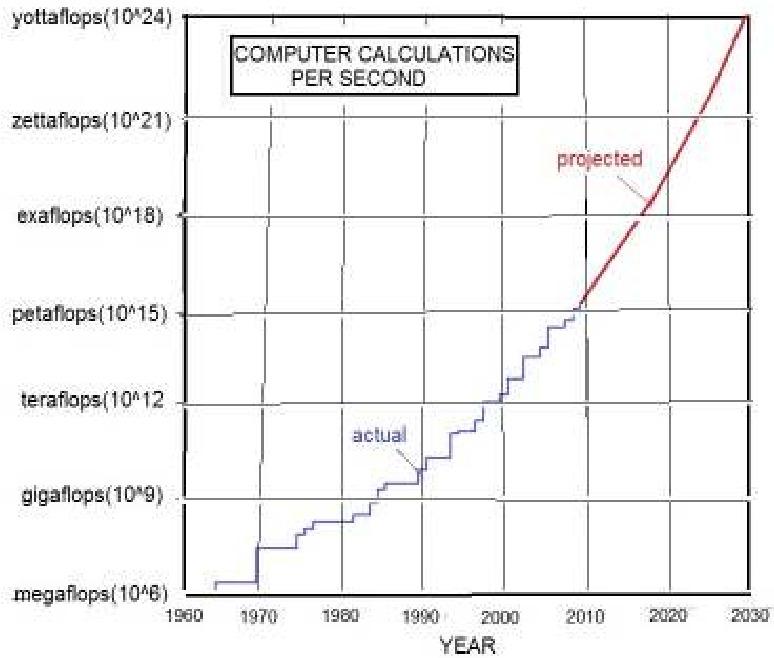
The concept of large and small is established by comparing the number to unity. For length the unit of length is the meter and time the second. There is a degree of anthropomorphism associated with these choices since the meter is comparable to the height of a human being and the second is about the time it takes for a human to respond to an external input. The original definition of the meter(m) was that it is one ten millionth of the quadrant distance from the earth's north pole to the equator. This unit of length has been refined over the years as first the length between two scratch marks on a metal bar stored in France to the more recent definition of the distance that light travels through vacuum in 1/c seconds, where c is the speed of light of 2.99792458 x $10^8$m/s. The original French definition in terms of quadrant length of the earth is still fairly accurate and lets one know at once that the circumference C of the earth is about 40 million meters. That is C=40,000 km=40Mm. The actual number is 40,008 km along a great circle passing through both poles. The time it takes for an astronaut to circle the earth in a near earth orbit is about $t=\sqrt{\frac{2\pi C}{g}}$ , where g=9.8 m/s$^2$ is the acceleration of gravity. That is, t= 5064.7s=1hr+24min. (I remember as an undergraduate back in October of 1957 going out in the early predawn hours to watch the first earth satellite(sputnik)pass overhead.) The roundtrip time it takes for a laser pulse to be shot from the earth to the moon and reflected back is about t=2.5 seconds**.** From this information one can determine that the distance to the moon is L=ct/2 or about 300,000 km=0.3 Gm. A year has 365.242199x24x60x60=3.155692599x$10^7$ seconds. That is 1yr=31.5569 Ms. Light travelling for one year will cover a distance of L=2.99792458x$10^8$x3.155692599x$10^7$ m=9.460528409 x$10^{15}$m=9.46petameters. This distance represents one light year. To get a feel for the magnitude of such a number it is interesting to note that our sun located at 93 million miles from earth is just a little over 8 light minutes away. The milky way galaxy, of which earth is a member, is about 100,000 light years across. It is estimated that the visible universe (as viewed from earth) has a diameter of 100 billion lightyears. This equals about 946 yottameters. The Big Bang is estimated to have occurred thirteen billion years ago, that is,13Gy ago.

Going the other way to smaller lengths than human dimensions, we first encounter the millimeter representing one thousands of a meter. This is about the smallest dimension required in the design and construction of things like clothing and houses. Next comes the micrometer($\mu$m) also referred to as the micron. Your eye will not be able to resolve this distance but it represents the dimensions of electronic circuit components in computers and cell phones. The wavelength of visible light lies at about 0.5 microns and this is about the limit of the smallest size etchings which can be produced on a silicon waver by chip lithography. Bacteria and viruses fall into the micron and sub-micron range. Even smaller length dimensions involve nano meters and are at the center of the present nanotechnology craze. A nano meter equals a billionth of a meter and falls into the dimension of atoms and molecules. The hydrogen atom has diameter of about one Angstrom=$10^{-10}$meter=0.1nanometer. A still smaller dimension is the diameter of a proton at about $10^{-15}$m=1 femtometer. Times associated with processes occurring at these smaller lengths will also be quite short relative to the human perception of time. For example Q switching using a laser cavity of length L yields pulses of time length of about $t=L/c=10^{-10}$ seconds in a 3cm long cavity. That is, light travels a distance of 0.03 meters in a tenth of a nanosecond. Another interesting short time interval is the time it takes a proton to make one complete circular orbit of the 27km circumference Hadron collider at CERN, Switzerland. Moving at near the speed of light such protons will take just $t= C/c =27 \times 10^3/2.99 \times 10^8=9.03 \times 10^{-5}$ s = 90.3μs to complete a circuit.

It should be pointed out that the above small and large number designations need not apply only to length and time. One sees the designation being used in conjunction with information storage, energy use, computer calculation speeds, etc. Here are some examples-

| | |
|---|---|
| Total World Information Storage by 2007: | $295 \times 10^{18}$ bytes=295exabytes |
| World Energy Consumption in 2008: | $474 \times 10^{18}$=474exajoules |
| Computer Calculation Speed by 2011: | $10.5 \times 10^{15}$ flops=10.5 petaflops |

An interesting sidelight is the increase in computer speeds with time. In the following graph I show the speed of electric super-computers (expressed in flops)-




The graph shows that it will not be anytime soon that one will be able to break semi-primes N of several hundred digit length into their components. When this does become possible then all public key cryptography becomes obsolete.

Numbers outside the range $10^{-24}<N<10^{24}$ also exist. They, however, have less significance as they are out of the range required for describing processes which directly involve man. In particular one has the googol , the Plank length, and the Dirac numbers. The googol and related numbers are defined as -

1 googol = $10^{100}$ 1 googolplex= $10^{\text{googel}}$ and 1 googolduplex= $10^{\text{googolplex}}$

These are really large numbers, especially the googolduplex which equals 10^10^10^100 where ^ is the usual symbol for exponentiation. It represents a good approximation to infinity and its inverse is a good approximation to zero. So far the googol and its related forms have not found any direct application although the name of the world's best search engine clearly has its name derived from this number. Next the Plank length is a number arising by looking for a length dimension involving the universal gravitational constant $G=6.673\text{x}10^{-11}m^3/kg\ s^2$, the Plank constant $h=6.626068\text{x}10^{-34}\ m^2kg/s$ arising in quantum mechanics, and c=2.9972458 m/s the speed of light. It is defined as-

$$l_p=\sqrt{\frac{hG}{2\pi C^3}} = 1.616199 \text{ x } 10^{-35} \text{ meter}$$

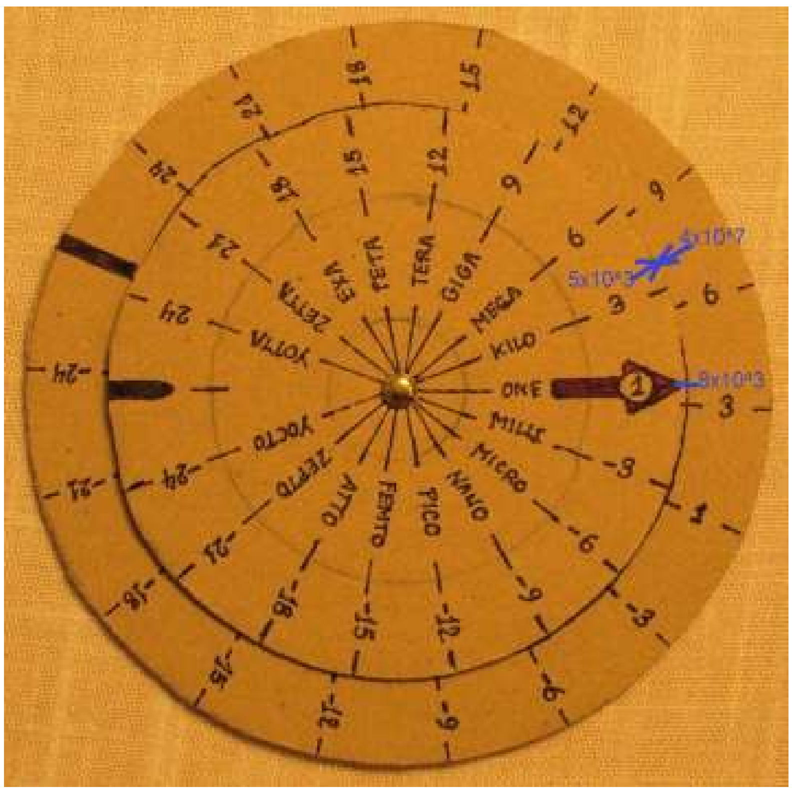
There exists no theory yet which combines gravitation with quantum mechanics and hence this length essentially has no physical significance but may in the future. Related numbers are the

Dirac numbers which have values of about $10^{40}$ . They are non-dimensional and relate gravitational with electrical and atomic quantities. One of these Dirac numbers is-

$$\frac{4\pi\varepsilon_o e^2}{Gm_p m_e} \approx 10^{40}$$

Here $m_p$ and $m_e$ are the mass of the proton and electron , e the charge of an electron, and $4\pi\varepsilon_o$ the permittivity factor.

Finally let us look at the following circular slide rule I have just constructed using two cardboard discs. It is capable of multiplying and dividing over the very wide range of 48 orders of magnitude and also helps in memorizing the names of large and small numbers. In the attached photo I show this slide rule as set for finding the speed of a satellite in near earth orbit. It involves aligning the value of the earth's circumference(C=40Mm) on the outer stationary scale with the known orbit period (t=5ks) on the inner movable scale. This yields the speed quotient of $(40/5)\text{x}10^3$=8km/s as indicated by the pointing red arrow.




Many of the aerospace engineers among you will recognize this speed as the familiar 26,000 ft/sec speed which John Glenn attained during the three orbits he took around the earth fifty years ago this month.