DETERMINING WHETHER A NUMBER IS PRIME OR COMPOSITE

A prime number is defined as any number N which is devisable only by itself and by one. Examples are 2, 3, 5, 7, 11, 13, 17, 19,…etc. Composites constitute all the remaining numbers . These are divisible by three or more integers such as 4, 8, 9, 12, 14, 15, 16, 18,…etc. It is an easy task to differentiate between composites and primes when N is small but becomes increasingly more difficult as N gets large. For instance, is N=5723086517 a prime or a composite? A simple mod test of N mod(6) says 5 which suggests it may be prime since it satisfies N=6n-1 and we know , from several previous articles, that all primes greater than three must have the form 6n±1. But since it is also possible for some composites to have the same form 6n±1, we cannot yet be certain that the N under consideration is prime. To find out if it is one could use a brute force approach which divides N by every prime less than sqrt(N) to see if any of the divisions yield an integer result, If so we have a composite. If not then N is a prime. This type of search is clearly a horrendous task and certainly impractical when N exceeds ten digits or mote. An alternate, much simpler route , is to make use of the point function-

$$f(N)=\frac{\sigma(N)-(N+1)}{N}$$

, where σ(N) is the divisor function of umber theory which sums up all divisors of N. We first discovered the function F(N) about five years ago. I term it the Number Fraction since it yields values in terms of rational fractions. The important point to note is that f(N)=0 denotes a prime while composites have f(N)>0. It is our purpose here to show how f(N) may be used to quickly find primes and in particular test the ten digit number N=5723086517 given above for primeness.

Although we can say that if σ(N)=N+1 or f(N)=0 is a necessary and sufficient condition that N is a prime, we choose here a more elegant approach not involving potential divisions by zero.

It is noted that when N is a prime p that –

$$f(p^2)=\frac{1}{p} \qquad and \qquad f(p^3)=\frac{(1+p)}{p^2}$$

Taking the ratio, we find-

$$1=p\{-1+\frac{f(p^3)}{f(p^2)}\}=-p+\frac{(1+p)}{pf(p^2)}$$

Canceling 1+p then leaves us with the important result that primes always satisfy-

$$1 = \frac{1}{pf(p^2)}$$

In view of this last result, we can now introduce a new point function-

$$F(N) = \frac{1}{Nf(N^2)}$$

One can call this as new Prime Number Function. It has value of one when N is a prime but less than one when N is a composite. The latter condition stems from the fact that $pf(p^2)<Nf(N^2)$ for non-prime N.

We can rewrite things in terms of the sigma function by noting that –

$$Nf(N^2) = \frac{\sigma(N^2) - N^2 - 1}{N}$$

It leaves us with the important new function-

$$F(N) = \frac{1}{Nf(N^2)} = \frac{N}{\{\sigma(N^2) - (N^2 + 1)\}}$$

Since the sigma function already exists in the repertoire of most computer math programs such as MAPLE, we prefer to evaluate FN) by the second form above.

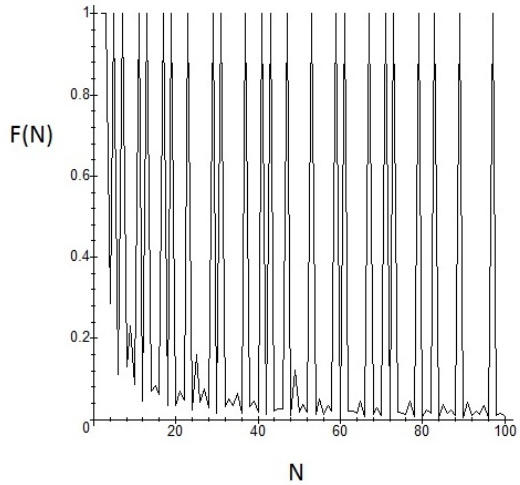
Let us now answer the question if N=5723086517 a prime. We find $\sigma(N^2)$ $=\sigma(32753719281067191289)=$ 32753719286790277807. So we find-

F(N)=(5723086517)/{ 32753719286790277807-32753719281067191290}=1

Hence N=5723086517 is definitely a prime.

The evaluation of F(N) is an extremely simple computational procedure as long as sigma(N^2) is known. We can easily generate a graph of F(N) versus N for any chosen range of N. For example, the following graph shows the first 25 primes appearing in the range 1<N<100 -

PRIME NUMBER FUNCTION F(N) SHOWING ALL 25 PRIMES PRESENT IN THE RANGE 2<N<100



F(N)=1 for primes

F(N)<1 for composites

The program used to obtain this graph was-

listplot(]seq([x,x/(sigma(x^2)-x^2-1)],x=2..100);

As N increases the density of primes to composite numbers decreases logarithmically. This is often called the Prime Number Theorem which says that the number of primes up to N goes as N/ln(N). This fact was already known to Legendre and Gauss who obtained the result by brute force counting. The result requires that N approach infinity. At N=100 one would estimate that the number of primes is 100/ln(100)=21.714 while as we have shown above it is actually 25.

We can readily determine the number of primes lying in the higher N range of 1000<N<1100 by use of the F(N) formula. A count using a F(N) versus N plot yields exactly 16 primes in this range. The prime number theorem predict a somewhat lower value of –

$$1100/\ln(1100)-100/\ln(1000)=12.309\ldots$$

Between N=1 million and N=1 million+100 , we find that the exact Formula F(N) shows there are 6 primes in the range and the prime number theorem predicts-

$$1000100/\ln(1000100)-1000000/\ln(1000000)=6.714$$

which is slightly higher than the exact value but approaching agreement. By deceasing the increment size of N one should be able to come up with a universal curve of F(N)=1 versus N far better than the Gauss n/ln(N) result or even the Riemann Li(N) result(see J.Derbyshire "Prime Obsession"). We leave this for a later note.

Let us get back to the F(N) function. We have shown that-

F(N)=1 means N is a prime and F(N)<1 implies we have a composite

There is no limit to the number size which can be tested by this rule provided our computer program is able to handle $\sigma(N^2)$. Lets try a few more numbers. Take first the large number-

N=818284590452353602874713526624977572470937631

Here we find F(N)=0.2327817898 x 10^{-44} . So we have a composite number without really knowing what the factors are. As a second comparable sized number look at-

N:= 8539734222673567065463550869546574495051

Here we find F(N)=1 and so it is a prime number. A third number is-

N=15690348530037422850799078491231511923072429075893

It yields F(N)=1 and so alsos prime. In doing the evaluation of F(N) for even larger N my computer has difficulty in quickly finding the value of $\sigma(N^2)$ unless F(N) happens to be one. This means I will typically terminate the calculations for large N if no answer is offered in a few second knowing we are dealing with a composite number. I then go on by increasing the number N several units until an answer of F(N)=1 is spit out. By this means we will know what value is prime and also know that the other cases requiring long calculation times are composites. Let us demonstrate this point by looking at the 95 digit long number –

N=:=48810057929746583976942766886963444693315023177984670142839108152807034998289726322756111320923

It is a composite because the F(N) calculation was taking a long time and thus terminated. However changing the last digit from 3 to 7 instantaneously produces the value F(N+4)=1 so that-

N=48810057929746583976942766886963444693315023177984670142839108152807034998289726322756111320927

Is definitely a prime number.

The present approach is far faster than other techniques for distinguishing primes from composite numbers. The approach may find application in certain areas of cryptography were it is thought that public keys consisting of the products of two large primes is essentially unbreakable.

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