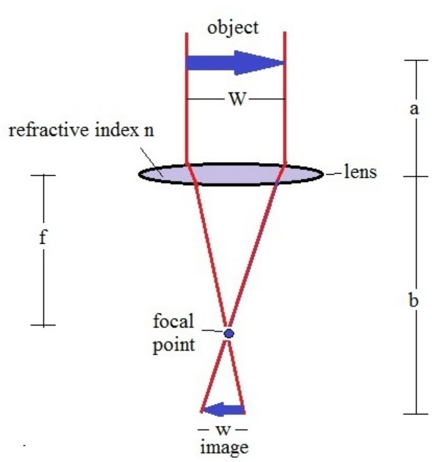
PROPERTIES OF LENSES

The lens is the an object central to the functioning of most optical systems. It first came into wide industrial use during the early sixteen hundreds in Holland for microscopes and eyeglasses (Lippershey, Leeuwenhoek), and also in England through Robert Hooke(an arch enemy of Newton), and by Galelio in Italy through his telescope. The existence of lenses of course also dates back to times much earlier than this considering the use of magnifying gems in ancient Rome and the existence of spectacle makers in Venice as early as 1400 AD. In another sense lenses have been around ever since life first developed here on earth. Consider the fact that every mammal is born with two fluid filled and adjustable focal length lenses at birth. These lenses sometimes need to be supplemented by the use of eyeglasses or even require lens replacement to maintain perfect vision. Out purpose here is to look at some of the more important properties of lenses.

A lens is essentially a transparent structure having two convex surfaces which bend incoming light rays following the familiar Snell's Law n sin(θ)=const . The material in the lens is homogeneous and should have an index of refraction n greater than that of the surrounding air. Glass and plastic are ideal lens materials although fluids like water may also be used. The surfaces of the lens are shaped so that two incoming parallel light rays converge to a focus point as shown-

LIGHT RAYS PASSING THROUGH A REFRACTIVE LENS



The light ray path is perfectly reversible so that the ratio of image to object width W can be larger, smaller, or equal to the inverted image width w. Every lens has its own built-in focal length f. From simple geometry using similar triangles and passing an extra ray through the lens center, we have the basic lens law -

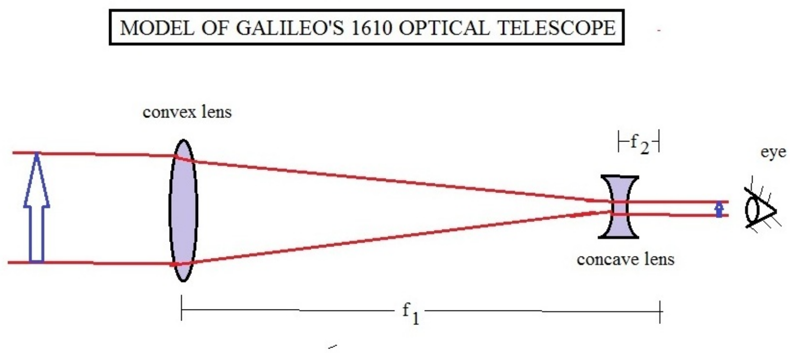
$$\frac{1}{a}+\frac{1}{b}=\frac{1}{f}$$

, where 'a' is the distance from the object to the lens center, 'b' the distance from the lens center to the image and f the focal length. The magnification is given by-

$$M = \frac{W}{w} = \frac{a}{b}$$

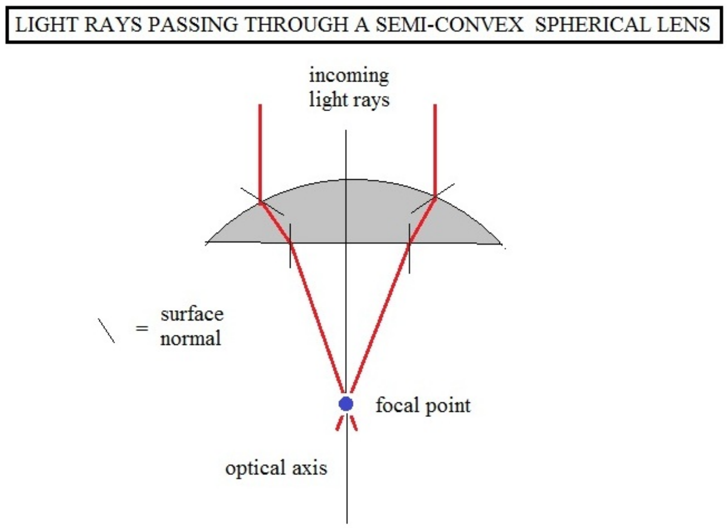
Note that the lens law is identical with the resistance law of two resistors in a parallel circuit. Sometimes the lens law is written with a minus sign in front of the 1/b term because of the way the distances a and b are measured. This can be very confusing to a college freshman , as it was for me some sixty years ago in my introductory physics course.

To make a simple telescope of the type Galileo constructed back in 1610 one needs only two lenses one convex of long focal length f_1 and a smaller concave lens of much shorter focal length f_2 as shown. The magnification will be equal to the ratio of the focal lengths. A schematic of his telescope looks as follows-



Since the second lens is negative, the first focal distance lies beyond the concave lens by length f_2. The advantage of this type of configuration is that the image will not be inverted. Modern binoculars and refractor telescopes use only convex lenses. Galileo ground his own lenses and adjusted their positions by trial and error. Microscopes work on essentially the same principle but the optics are reversed. Cameras and the human eye use only one convex lens in their optics. Two eyes, just as two ears per person, are there to allow a distance and direction measure of incoming signals. The recent interest in micro-cameras such as needed for I-phones , mini-drones, and certain spy equipment have greatly increased our ability to manufacture very precise micro-lenses. Many of these lenses are smaller than even Leeuwenhoek’s 1.5 mm diameter spherical lenses used in some of his many microscopes. Some of his best single lens microscopes had resolutions down to one micron. Since the wavelength of light lies in the 0.4 to 0.6 micron range, diffraction effects will not hinder the progress in electronic micro-cameras from going to still smaller scales than at present.

Let us next look at the details of the path of two parallel light rays impinging on and then exiting a plano-convex lens. To keep the calculations simple we assume the lens to have its upper surface defined by the off-center circle $x^2+(y+0.8)^2=1$ with $y\geq 0$. The lower flat surface is given by the straight line y=0. Two light rays enter from the top and are parallel to the optical axis. A schematic of the set-up looks as follows-



Let us now let the right ray impinge on the lens at $x=x_0$ and $y=y_0$. According to Snell's Law the ray will bend according to-

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

,where the ns are the indexes of refraction of the outside air and the assumed crown glass lens and the θs the angle the light ray makes with respect to the surface normal before and after refraction. The law is easily established by variational principles and says essentially that a light ray will be bent toward the surface normal when entering a higher index of refraction material. The index represents the ratio of the speed of light in a vacuum compared to that in the lens medium. For crown glass n=1.5 , for water it is n=1.333 and for air it is very close to n=1. Diamond has one of the highest known coefficients of refraction at about n=2.4.

The surface normal for the ray at the right entering the lens at $[x_0,y_0]$ has the positive slope –

$$\frac{dy}{dx} = \frac{y_0 + 0.8}{x_0}$$

Therefore the angle of incidence θ_1 becomes-

$$\theta_1 = \frac{\pi}{2} - \arctan\left[\frac{(y_0 + 0.8)}{x_0}\right]$$

By Snell's Law the refractive angle becomes-

$$\theta_2 = \arcsin\left[\frac{\sin(\theta_1}{n}\right]$$

From the lens geometry we can also see that the refracted ray will impinge on the lower flat interface at $[x_1,0]$ where-

$$x_1 = x_0 - y_0 \tan(\theta_1 - \theta_2)$$

Applying the Snell's Law again then shows that-

$$\sin(\theta_3) = n \sin(\theta_1 - \theta_2)$$

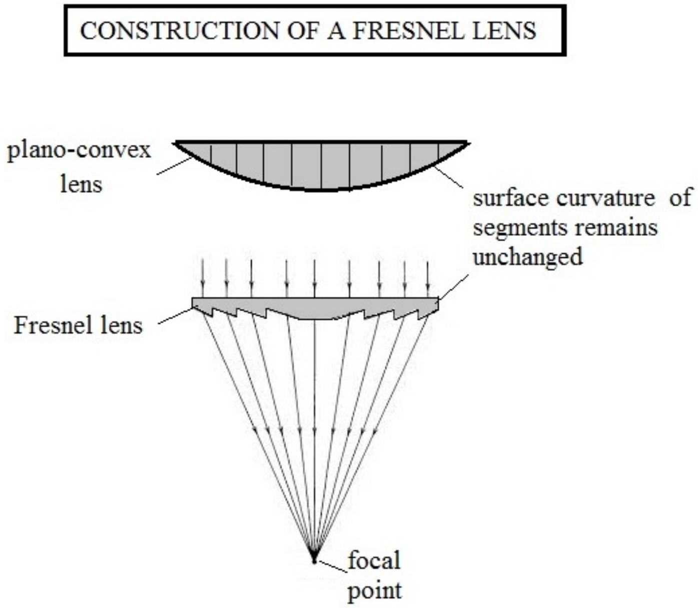
We can now combine all of the above equations to show that the lens has a focal length of-

$$f = x_1 \cot\{\arcsin[n \sin(\theta_1 - \theta_2)]\}$$

This focal length will vary slightly as the values of x_0 are changed . This variation is known as spherical aberration and can be corrected by slightly regrinding the lens surface or using a Schmidt Plate in front of the lens.

Let us determine the focal length for a ray hitting the upper surface at $x_0=0.3$ where $y_0=0.1539$. Also let n=1.5 which corresponds to crown glass. We find $\theta_1=1.0044$ rad , $\theta_2= 0.59748$ rad and $\theta_3=0.6356$ rad. These in turn lead to $x_1=0.2337$ and finally we have a focal length of f=0.3167. Since the lens here has a width of 1.2, the focal length equals approximately one quarter of this. You will notice that all focusing ability will be lost when n=1 for then there will be no refraction. For mammal eyes the fluid in the cornea lens has an index of refraction of about n=1.38 which is just slightly above that for water. Cataracts form in many older individuals when this fluid becomes cloudy. Total blindness is a possibility if untreated.

An obvious observation from the above ray tracing result is that bending of light rays occur only at the air-glass and glass-air interfaces and that the rays follow straight lines within the lens itself if the glass is homogeneous. This suggests an idea first put into practice by the French physicist Augustine Fresnel (about 1822) that one can design some new large lenses requiring only a minimum of glass. The resultant lenses received immediate application for lighthouses throughout the world. The essence of a Fresnel lens is made clear in the following schematic-



We show here a standard plano-convex lens. It is cut into equal width slabs as shown with the slabs being moved down by different amounts to form a thin plate where the bottom surfaces match exactly the curvature of the original plano-convex lens. The result is a Fresnel lens. It contains considerable less material than the original lens, yet is able to produce a reasonable focus good enough for many applications such as for search lights and for producing concentrated solar energy. These lenses have found wide application in recent years in connection with solar energy applications. The lenses are cheap to produce using plastic sheets ($n\approx1.4$) on which are imbedded the grooves. A draw back of the use of plastic lenses for solar applications is that they rapidly degrade in sunlight. This suggests it might be a better idea to construct some Fresnel mirrors where the grooves are electroplated with silver. The use of 3D printers for producing smaller Fresnel mirrors might also prove beneficial.

To complete our discussion on lenses, let us work out the relation between surface slope of the grooves as a function of distance from the optical axis, required focal length, and index of refraction of material used for a Fresnel lens. In our model the lens consists of multiple different width grooves at different distances from the optical axis. A light ray coming in parallel to the optical axis is unaffected by the upper flat surface but will refract at the groove on the other side following Snell's Law. To simplify things let the groove bottom not be curved but rather take the form of a straight line with slope $\tan(\varphi)$. After some manipulations, we find that the light ray coming from inside the lens will bend toward the optical axis and intersect this axis at a focal distance –

$$f = x\cot\{\arcsin(n\sin\varphi)-\varphi\}$$

Here x is the distance from the inclined groove bottom to the optical axis, n the index of refraction of the material, and φ the slope of the interface. So for example, if we look at the Fresnel ring at x=10cm, have n=1.5, and demand that f is also 10 cm, the grove bottom angle becomes-

$$\varphi=0.7282\ \text{rad}=41.726\ \text{deg}$$

If we halve x to 5cm but keep f and n as before, we find the shallower incline angle of-

$$\varphi=0.63609 \text{ rad}=36.4457 \text{ deg}$$

For any slope exceeding $\varphi=\arcsin(1/1.5)=0.7297$ rad= 41.81 deg the light ray will be internally reflected and no longer contribute to the focusing power of a Fresnel lens. Such a problem will not occur for Fresnel mirrors. It should be possible to construct some good Fresnel mirrors by laying out a long strip of silvered mylar in the form of a spiral and then progressively inclining the strip as one moves further out from the optical axis.