WHAT IS A LOGARITHM AND HOW CAN IT BE USED TO RAPIDLY MULTIPLY AND DIVIDE NUMBERS ?

The logarithm of a number is the power p to which a base b is taken in order to match the number. That is-

$$N = b^{p} \quad where \quad p = \log_{b} N$$

The concept was first developed in 1614 by John Napier(1550-1617) of Scotland and the then expanded into the practical application of slide rules by Henry Briggs, Edmund Gunter, and Wiliam Oughtred of England later in the sixteen hundreds. What Napier noticed (and today every school child knows) is that the product of two numbers and their quotient may be written as-

$$N\,x\,M = b^{[\log_{b} N + \log_{b} M]} \quad and \quad \frac{N}{M} = b^{[\log_{b} N - \log_{b} M]}$$

Thus, if one knows the logarithms of the numbers N and M, then taking the inverse values yields their product and/or quotient. There are just two bases b which are in common use. The first of these is the base b=e=2.718281828459045… which produces the natural logarithm of a number denoted by ln(N). The second base is b=10 and it produces the Briggs logarithm log(N). Extensive logarithm tables have been prepared using both of these number basis. Today these tables are obsolete due to the advent of the electronic computer. Also the logarithm of a number , if needed, can now be readily obtained with a hand calculator.

Let me give you a couple of examples involving calculations with logs. Consider first the product of N=2578 and M=8239. We have, with aid of our hand calculator, that-

$$2578x8239 = 10^{[3.4112829+3.9158745]} = 10^{7}x10^{0.3271574} = 21{,}240{,}141.22..$$

The actual value is 21,240,142 so the logarithm approach will generally only be as accurate as the number of digits appearing in the log. In the calculation we used only the first seven digits in the logarithm. As another example we carry out the operation-

$$\sqrt{2696}\cdot 75623/435^{6}=\frac{10^{3.4307198/2}10^{4.8786539}}{10^{(2.638489)6}}=10^{-9.23692}=5.795354..\cdot 10^{-10}$$

My hand calculator yields 5.7953317x10^{-10} for this operation.

One can look at mathematical manipulations using logarithms as an analog approach as opposed to the standard digital approach of carrying out the operation on the original numbers by hand or with an electronic computer. Sometimes the answers with logarithms will be exact such as for example-

$$64x256/\sqrt{1024}=2^{6}x2^{8}/2^{5}=2^{9}=512$$

where we have used the base b=2.

To convert from a number's logarithm in base b=e to any other base b, one simply uses-

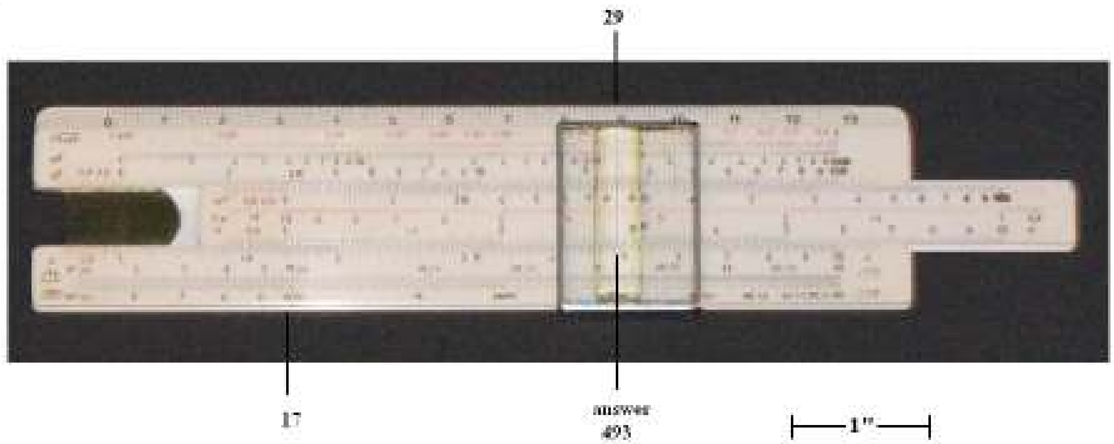
$$e^{\ln(N)}=b^{\log_b(N)}\ \textit{or the equivalent}\ \ \ln(N)=\ln(b)\log_b(N)$$

For the base b=10 we have ln(N)/log(N)=2.30258509299404568401799…

The logarithm approach to multiplication and division is generally much faster than carrying out calculations by hand and thus found extensive use in the aviation, automotive, and military industries in the early part of the 20th century before the advent of electronic computers.

An important application of logarithms was the slide rule. It was used extensively by mathematicians, physicists and engineers until about 1974 when cheap hand calculators became available. Today slide rules are a historical curiosity and many of you will not be familiar with this handy calculating device. I show you here my own ivory slide rule brought back from Europe by my father and given to me as a gift when I was in tenth grade. I have set it to show the multiplication of 17x29=493 -

SLIDE RULE USED DURING MY HIGH SCHOOL, UNDERGRADUATE, AND GRADUATE EDUCATION
(Slider is set to show the multiplication 17 x 29 = 493)



It consists essentially of some stationary logarithmic scales plus a central sliding scale along which the integers are marked in a logarithmic manner. In such a device multiplication and division of numbers is replaced by addition and subtraction of their logarithms and the answer can be read off directly helped by the magnification glass shown. Such hand slide rules can get results accurate to about three places which in many instances is perfectly sufficient. Most of the mathematical calculations required in my PhD dissertation were done with this slide rule. Other scales on the slide rule shown allow squaring, cubing, inverting, plus trigonometric calculations. I remember in college larger versions of these slide rules being carried around campus attached to the belts of engineering majors. Today I see similar displays except this time its iphones with wifi internet capability hanging from their belts. As the French say “plus ca change, plus c’est la meme chose”