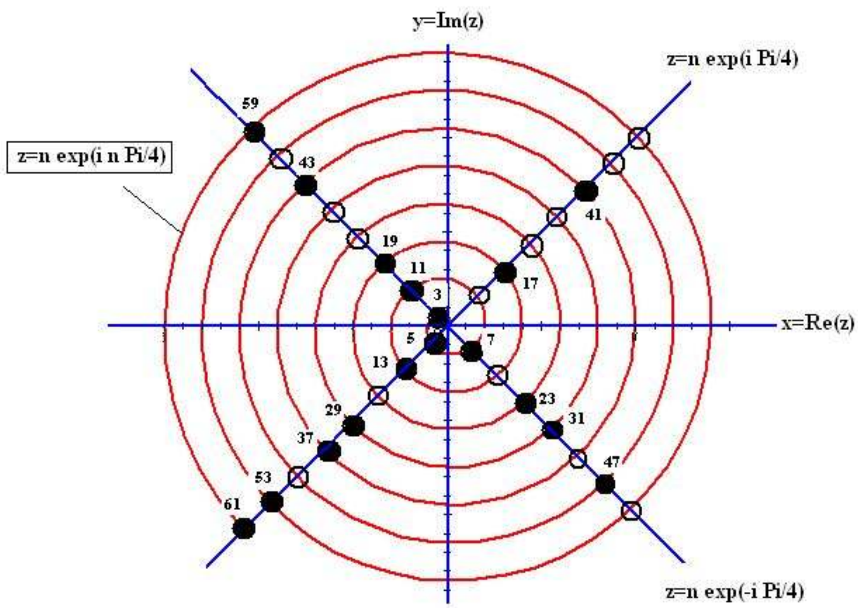
MERSENNE NUMBERS AND THE INTEGER SPIRAL

Several years ago we found a new way to plot numbers using an Integer Spiral where all positive integers n correspond to the intersection points of the Archimedes spiral z=n exp(iπn/4) and the radial lines $\theta=k\frac{\pi}{4}, where\ k=0,1,2,3,4,5,6,7$. The interesting thing about this representation is that all odd numbers lie along the four possible diagonal lines and hence that all prime numbers(with the exception of 2) must lie along these same diagonals. Here is a picture of the odd integers as they are found along the Integer Spiral-



I have marked the odd integers which are prime by solid black circles. The even integers are not shown but all lie along the x or y axes. For example the number n=40 lies along the positive x axis at the fifth turn of the spiral. The odd numbers along the diagonals are separated from each other by exactly 8 units. As we have shown in an earlier note , the well known Ulam Spiral can be readily morphed into the present Integer Spiral and, hence, the prime number patterns seen in the Ulam Spiral indicate no more than that all primes above n=2 are necessarily odd numbers. <u>It has no predictive capabilities</u> concerning whether an odd number is prime or composite.

What is of interest to us in this note is the location of a special subclass of integers , namely, the Mersenne Numbers M(n)=2^{2n+1} -1 , n=1,2,3,4,… . Some of these numbers are prime numbers while the majority are composite. The following table gives the values of the first 15 Mersenne Numbers-

n	$M(n)=2^{2n+1}-1$
1	7
2	31
3	127
4	511
5	2047
6	8191
7	32767
8	131071
9	524287
10	2097151
11	8388607
12	33554431
13	134217727
14	536870911
15	2147483647

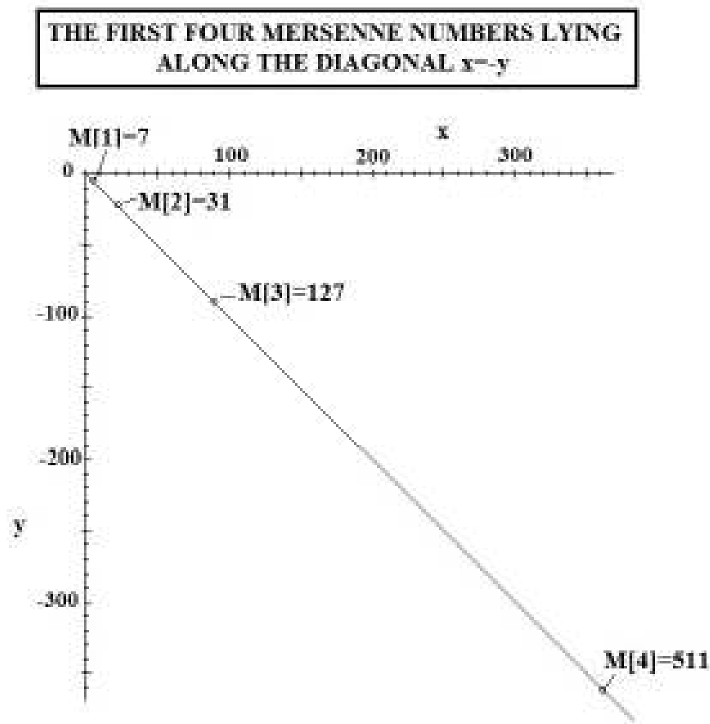
Those of these numbers which are prime are indicated in red. One can generate any desired larger Mersenne Number via the formula-

$$M[n+k] = 2^{2k}(M[n]+1) - 1$$

where k=1,2,3,4… . Thus $M[16]= 2^{30}(7+1)-1= 8589934591 = 7(23)(89)(599479)$ and

$M[53]=2^{107}-1= 162259276829213363391578010288127$

This last number is indicated to be prime by my home PC using a Lucas-Lehmer primality test. Notice that all M[n]s are separated from each other by 8 times a constant and all fall along the same diagonal line in the fourth quadrant of the x-y plane as shown-



Note that M[2]-M[1]=3(8), M[3]-M[2]=12(8), and M[4]-M[3]= 48(8). From this we see that the separation between neighboring M[n]s increases with increasing n as-

$$\Delta M=M[n+1]-M[n]=3(2^{2n+1})$$

The separation shows a dramatic growth with increasing n. To determine which of these Mersenne Numbers M[n] is prime we can use the MAPLE probalistic operation isprime(M[n]); Looking at the first one hundred values of n, we find the following 11 primes-

M[1]=7

M[2]=31

M[3]=127

M[6]= 8191.

M[8]= 131071

M[9]= 524287

M[15]= 2147483647

M[30]= 2305843009213693951

M[44]= 618970019642690137449562111

M[53]= 162259276829213363391578010288127

M[63]= 170141183460469231731687303715884105727

In addition we find M[260], M[303], and M[639] to be prime when considering all M[n] up to n=1000. What is noted is that all M[n[, be they prime or composite, always end in the integer 1 or 7 when expressed as base 10 numbers or will always have a binary form consisting only of all ones. Thus in binary we have-

M[10]= .1111111111e21 , M[100]= .1111111111e401 and M[1000]= .1000000000e4002

Note the very large increasing size of M[n] as n is increased. Also one notes that for all the prime M[n]s we have found up to n=1000 , the quantity 2n+1 is also a prime. So, M[639]=2^{1279}-1 has 1279 as a prime. However a prime power of 2 need not necessarily guarantee a prime M[n]. For example, M[11]=2^{23}-1 =47 x 178481 is composite.

People are still pursuing finding ever larger Mersenne primes. The latest record is the 47th Mersenne Prime M[21556304]. It is 10^7 digits long. The search for still larger Mersenne Primes continues. It still is not known whether the number of Mersenne Primes is finite or infinite. What is clear at the moment is that the difficulty in finding the next highest prime is a formidable task and one never knows whether the next highest M[n] prime actually does exist. It should be remembered that the Fermat Primes F[n]= $2^{2^n}+1$ only exist for n=1, 2, 3, and 4. No Fermat Primes above n=4 have ever been found.

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