ON THE COMPLETE GOLDBACH CONJECTURE

It is well known from the Goldbach Conjecture that any even integer may be represented as the sum of just two primes provided the primes are three or greater. We recently confirmed this point by noting that even integers can be represented by multiple Goldbach pairs whose number goes up approximately as –

$$GP \approx 0.3N^{0.7}$$

(see http://www2.mae.ufl.edu/~uhk/GOLDBACH-PAIRS-NEW.pdf)
Since the Goldbach Conjecture for even nukmbers N requires only one prime pair, it is clear that the conjecture is correct.

We address here the second and less well known part of his conjecture that all odd numbers can be represented by the sum of at most three distinct primes To show that this is also correct, we start with a quick listing of the first 50 primes . They are-

primes={2,3,5,7,11,13,17,19,23,29.31,37,41,47,53 ,59,61,67.71,73,79,83,89,97,101,103, 107,109,113,127,131,137,139,149,151,157,163,167,173,179,181,191,193,197, 199,211,223,227,229}

We next make a list of the first N odd integers and see how they may be presented by the sums of some of the above primes excluding 2. Here is a table starting with N=7-

7=7	**31=31**	**55=7+11+37**
9=3+3+3	**33=3+11+19**	**57=3+7+47**
11=11	**35=5+7+23**	**59=59**
13=13	**37=37**	**61=61**
15=3+5+7	**39=3+7+29**	**63=3+7+53**
17=17	**41=41**	**65=5+7+53**
19=19	**43=5+7+31**	**67=67**
21=3+7+11	**45=3+5+37**	**69=3+7+59**
23=23	**47=47**	**71=71**
25=3+5+17	**49=5+7+375**	**73=73**
27=3+5+19	**51=3+7+41**	**75=3+11+61**
29=29	**53=53**	**76=5+29+31**

The primes are marked in red. The triplet primes shown are typically just one of many for a given odd N. All the triplets shown, when N is not a prime, are indeed represented by the sum of just three distinct primes. Except for 9, the prime expansions contain no duplicate primes. To further support this observation, we look at the larger odd number N=6751. Here the nearest prime lies 14 points below at 6737. So that 6751=3+11+6737 . Take next the even larger number N=568427 . This may be re-written as 568427=5+31+568391. So again the expansion just equals the sum of three separate primes. A still larger odd integer is N=1573289052457. Here we again find a three prime sum-

1573289052457=7+41+1573289052409

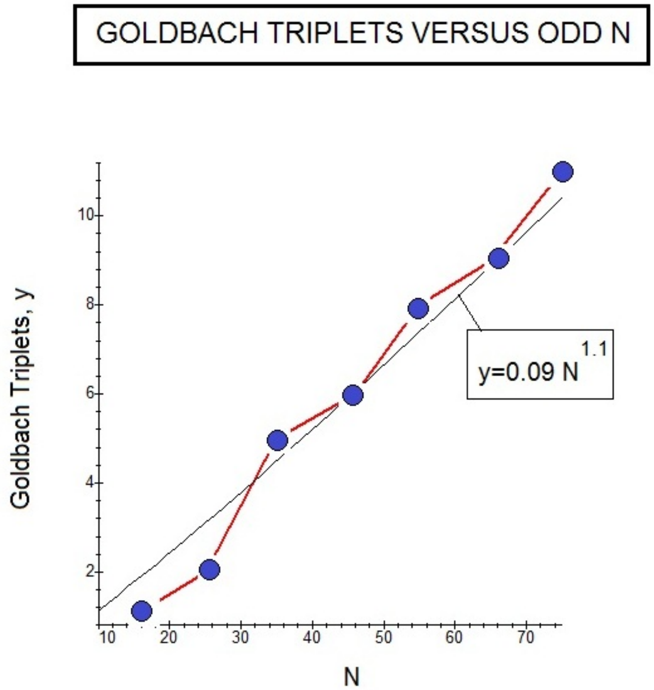
By reducing the last integer by one unit we get an even number which according to Goldbach;s Conjectrure indeed can r represented by the sum of just two primes as shown-

1573289052456=5+1573289052451

To prove that the second part of the Goldbach Conjecture holds for all odd N we construct a second table in which we represent the number of possible Goldberg Triplets for a given odd N for several different Ns-

N	Goldberg Triplets	Number of Triplets not containing 2, repetitions and interchange
15	3+5+7	1
25	3+5+17, 5+7+13	2
35	5+7+23, 5+1+19, 3+13+19, 5+13+17, 7+11+17	5
45	3+5+37, 3+11+31, 5+11+29, 3+13+29, 5+17+23, 3+19+23	6
55	3+5+47, 3+11+41, 7+11+37, 5+13+37, 5+19+31, 7+17+31, 3+23+29, 7+19+29	8
65	5+7+53, 5+13+47, 7+11+47, 5+19+41, 7+17+41, 5+23+37, 11+17+37, 5+29+31,11+23+31	9
75	3+5+67, 3+11+61, 5+11+59, 3+13+59, 3+19+53, 5+17+53, 5+23+47, 11+17+47, 5+29+41, 3+31+41, 7+31+37	11

What is clear from this last table is that the number of Goldbach Triplets for odd integers N increases monotonically with increasing N. Here is a picture of this behavior in the range examined-



So like in the even N cases were the number of Goldberg doublets increases with N, the increase in triples here means that the second portion of the Goldberg Conjecture is also correct since one requires only a single triplet for a given odd number for the conjecture to be valid.

One can summarize the complete Goldberg Conjecture as-

Any even integer can be represented as the sum of two primes and any odd integer can be represented by the sum of three primes.

In this conjecture one neglects the first prime at p=2 and takes the primes to be different from each other.

Here is a brief table working the Goldbach Conjecture backwards-

EVEN N	ODD N
3+5=8	3+5+7=15
11+19=30	5+11+17=33
29+59=88	13+19+29=61
41+73=114	59+61+67=187
199+229=428	163+193+211=567

Note that the sum of four primes would produce an even number and five primes an odd number, again excluding p=2. For semi-primes where N=pq we find N to be an odd integer but S=p+q an even integer. Since S also equals sigma(N)-N-1 we have that the divisor function sigma(N) is even for semi-primes. For example N=533 has p=13 and q=41. It is odd while S=54 is even as is sigma(533)=588.

U,H,Kurzweg
August 24, 2018
Gainesville, Florida