PROPERTIES OF MULTI-PRIMES

It is well known from number theory that any positive integer \( N \) can be expressed as the product of primes taken to specified powers. That is:

\[
N = \prod_{k=1}^{m} (p_k)^{a_k} \]

In studying this expansion in some detail we recently observed that if the expansion does not contain \( p_1 = 2 \) and \( p_2 = 3 \), that there exists a new sub-class of numbers \( M \) with the interesting property that any number of the form:

\[
M = \prod_{k=3}^{m} (p_k)^{a_k}
\]

satisfies \( M = 6n+1 \) or \( M = 6n-1 \), where \( n \) is a given positive integer. In terms of modular arithmetic we have \( M \mod(6) = 1 \) or \( M \mod(6) = 5 \). Thus, for example,-

\[
M = 7 \times 7 \times 17 \times 61 = 81923 = 6(13654) - 1
\]

and-

\[
M = 17 \times 41 \times 127 \times 223 = 19739737 = 6(3289956) + 1
\]

In terms of modular arithmetic one can say that-

**Any integer of the form \( M \) has the property that it satisfies \( M \mod(6) = 1 \) or \( 5 \).**

It is our purpose here to examine the numbers \( M \) in greater detail.

First of all we notice that \( M \mod(6) = 5 \) is equivalent to \( M \mod(6) = -1 \) indicating we are dealing with a cyclic process in which all integers \( N \) (and not just the \( M \)s) fall somewhere along six possible radial lines \( 6n, 6n+1, 6n+2, 6n+3, 6n+4, \) and \( 6n+5 \) in the polar plane at a point \( [r,\theta] = [N, N\pi/3] \). Drawing a listplot of all positive integers five or greater we get the following hexagonal pattern:
One sees that the graph represents all positive integers N including the sub-class M. The latter numbers N greater or equal to five clearly lie only along the radial lines 6n±1. The coordinates of all positive integers N are located in polar coordinates at-

\[ [r, \theta] = [N, N\pi/3] \]

We have typed in the numbers at their unique locations. A number such as M=35 lies at a radial distance of r=35 from the origin at angle \( \theta = 35\pi/3 \approx -\pi/3 \), which means along 6th turn of the hexagonal spiral.

All prime numbers five or greater, which form a further sub-class of the Ms, clearly lie only along the radial lines 6n±1. Reading off the 6n+1 line we see the primes 7, 13, 19, 31 and along the 6n-1 radial line we have the primes 5, 11, 17, 23, 29. Notice the gap at 25 and 35. These represent semi-primes 25=5x5 and 35=5x7. As we have already noted in several earlier papers, the spacing between primes along a radial line is always a multiple of six. That is, the prime M=61 leads to another primes at 67, 73, and 79 but fails at 85=5x17 and 91=7x13 and then continuous on at 97, 103, and 109.

Mathematicians have for years tried to make sense of the location of primes along an Ulam Spiral. We first pointed out back in August of 2008 that such a search for the meaning of such a pattern is futile since a simple morphing procedure just shows that the prime number
locations represent no more than a statement that all primes above three have the form 6n±1 (see http://www2.mae.ufl.edu/~uhk/MORPHING-ULAM.pdf)

Now getting back to the M numbers. If we have a semi-prime of the form M=6k+1=pq where p and q are primes, it must be true that p=6n+1 and q=6m+1 or p=6n-1 and q=6m-1. Take the semi-prime M=1357=6(226)+1. There p=6(4)-1=23 and q=6(10)-1=59. If M=6k-1 we expect p=6n-1 and q=6m+1.

To test whether or not M is a prime one first checks if M=5+6n or M=7+6n for any integer n. If so, one next uses the concept of a number fraction f(M) to see if the number is prime or composite. We have defined the number fraction in earlier articles as-

\[ f(M) = \frac{\sigma(M) - M - 1}{M} \]

, where \( \sigma(N) \) is the divisor function of number theory. When f(M) vanishes then M is a prime, if it has a small value just slightly above zero it is likely to be a semi-prime. For values above zero but less than one it is likely to be a higher multiple-prime number. Let us run a test to check this. Consider the seven digit number M= 2756203 which has M mod(6)=5 and can be written as 6(46934)-1. So it is an M number and could be a prime. Next working out f(M) we get f(M)= 0.0066773… which is just slightly above zero meaning that M is not a prime but likely will be a semi-prime. Indeed using the computer operation ifactor we find M= 2756203=151x18253 and so it is a semi-prime. Consider another number-

\[ N=398621801 \] which has \( N \mod(6)\)=5

Making it an M number. It can be written as 6(66436967)-1. To test if it is prime we evaluate f(N) to find f(398621801)=0. Hence it is a prime number.

Any number where \( N \mod(6)\) =3 can never be a true M multiple prime since it will contain the primes 2 and 3. Look at the number-

\[ N=98163759324981 \] which has \( N \mod(6)\)=3

It is thus clearly not an M number since its factor contains three and hence cannot be a prime or semi-prime number either.

To test whether any number pair N-1 and N+1 is a twin prime we can write-

\[ (N - 1)(N + 1) = N^2 - 1 = 36n^2 \]

This reduces to-

\[ N = 6n \]
for any integer \( n \) which makes both \( N \pm 1 \) a prime. Here is a table for the first few twin primes:

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N-1 )</td>
<td>5</td>
<td>11</td>
<td>17</td>
<td>29</td>
<td>41</td>
<td>59</td>
</tr>
<tr>
<td>( N+1 )</td>
<td>7</td>
<td>13</td>
<td>19</td>
<td>31</td>
<td>43</td>
<td>61</td>
</tr>
</tbody>
</table>

Note that the numbers \( n=4, 6, 8, \) and \( 9 \) do not lead to twin primes since either one or the other forms \( N \pm 1 \) are composite. For \( n=2090 \) we find the twin primes 12539 and 12541.

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