RELATION BETWEEN NODE NUMBER , CONNECTORS , AND AREAS CREATED IN 2D

It is well known since the time of Euler that there exists a universal relation stating that-

V+F-E=2

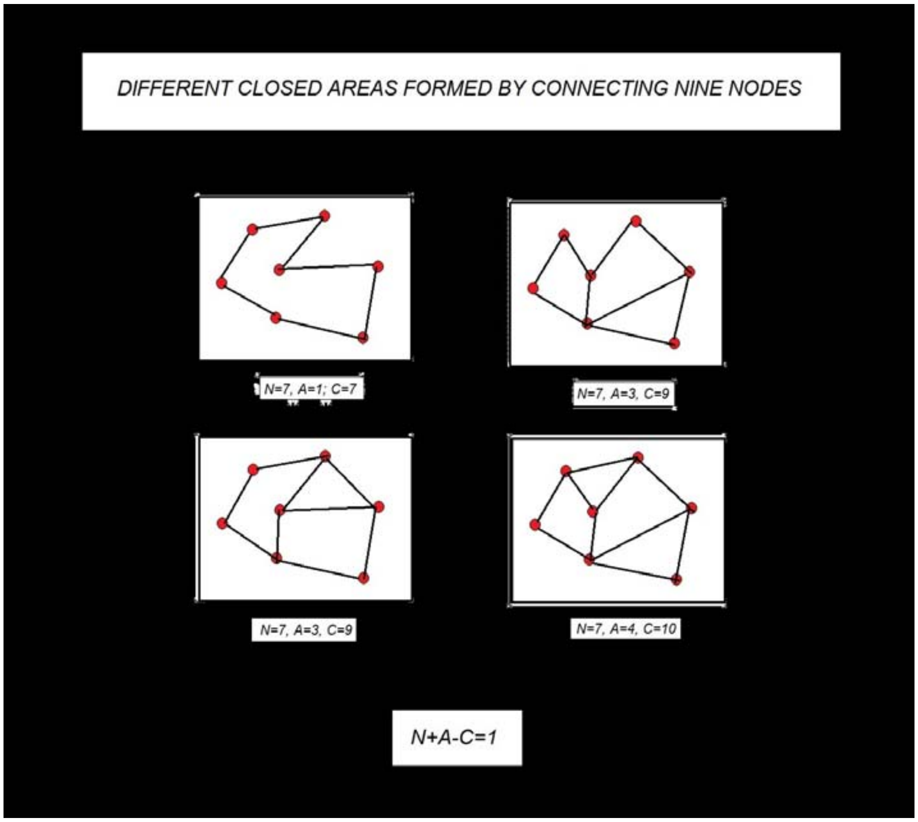
, where V represents the number of vertexes of a 3D polyhedron, F the number of its faces, and E the number of edges. For a cube one has V=8, F=6, and E=12. Thus the Euler formula yields 8+6-12=2. Moving things to 2D takes one into the realm of graph theory. There the usual statement of the Euler formula reads-

N+R-A=2

, where N represents the number of nodes, R equals the number of regions present including the region outside a closed node structure, and A the number of interior angles present. Thus for a hexagon we have N=6, R=2, and A=6. This produces 6+2-6=2 in agreement with the above formula.

Our purpose here is to consider a related problem in which we start with N points (termed nodes in graph theory) lying within the x-y plane and then connecting these to each other by C connectors to form a closed area composed of A sub-areas.

To find out the relatiuon between N,A, and C, we start with the following graph in which one has nine randomly placed nodes N=9 connected to each other by a different number of connectors C to form A sub-areas-



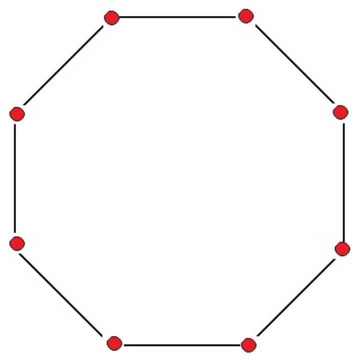
We note that in each of the different connection patterns, it is always true that-

$$N+A-C=1$$

regardless what type of sub-areas the figure has. By replacing A by A+1 one in effect recovers the Euler result for 2D.

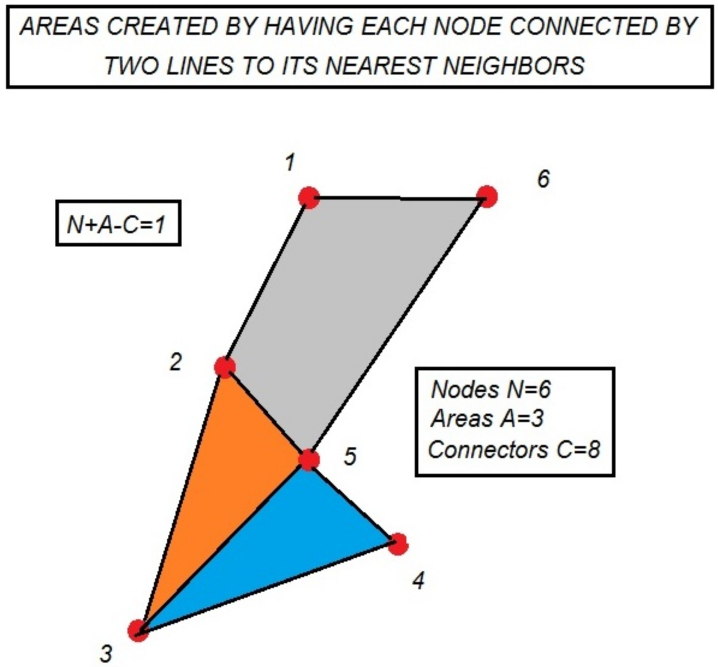
When the nodal point are placed at the vertexes of an N sided regular polygon and connected to their two nearest two neighbors one generates the perimeter of the polygon. For an octagon we have A=1, N=C=8. Here is its graph-

REGULAR OCTAGON WITH N+A-C=1



Nodes-8, Connectors=8, Areas=1

A more complicated configuration constructed from six randomly placed nodal points is the following-

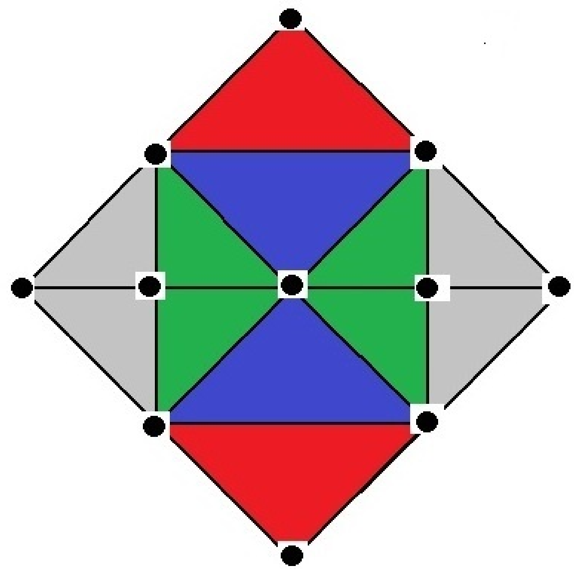


Here we have six nodes and eight connectors forming two triangle sub-areas and one quadrangle sub-area. That is N=6, A=3, and C=8. This again obeys the law-

$$N+A-C=6+3-8=1$$

This law continues to hold no matter what the number of points in the 2D plane one is dealing with. Here is a pattern for eleven nodal points-

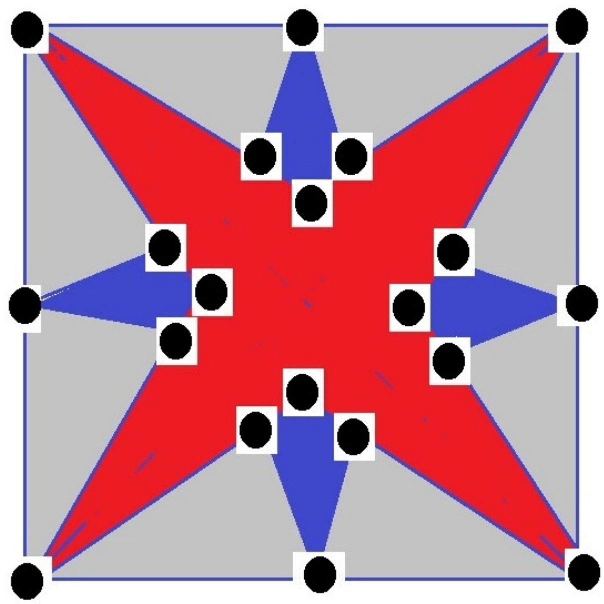
PATTERN GENERATED BY 11 NODAL POINTS AND TWENTY-TWO CONNECTORS



N=11, A=12,C=22

As a final 2D pattern we look at the star configuration produced by 22 nodes placed symmetrically about both the x and y axis. Here is the graph-

STAR PATTERN PRODUCED BY TWENTY NODES AND THIRTY-TWO CONNECTORS



N=20, A=13 , C= 32

There we have a total of A=13 sub-areas (grey, red, and blue) . The connectors add up to C=32. Again we have-

$$N+A-C=20+13-32=1$$

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