A NEW TEST FOR PRIMES

In several recent notes we have looked at a type of prime number defined by-

\[ Q = 6n \pm 1 \quad \text{with} \quad n = 1, 2, 3, \ldots \]

and found that these Qs can represent all primes greater than 3. Thus, we have the primes-

\[
2574983 = 6(42914)-1, \quad 63799137 = 6(10633190)-1 \quad \text{and} \quad 763035523 = 6(127172587)+1
\]

However, there are also many Qs which do not correspond to prime numbers such as-

\[
138941 = 6(23157)-1 \quad \text{and} \quad 579331093 = 6(96555182)+1
\]

How does one distinguish between the two groups? The answer is that one needs to use the number fraction \( f_N \) (see our earlier note at [http://www2.mae.ufl.edu/~uhk/NUMBER-FRACTION.pdf](http://www2.mae.ufl.edu/~uhk/NUMBER-FRACTION.pdf). It will be different from zero for Ns which are composite but vanish when N is a prime. Thus we have the new prime number test -

**Every prime number above 3 has \( N \mod(6) \) equal to 1 or 5 and in addition it must satisfy the condition \( f[N]=\frac{\text{sum(divisors of N)}-(N+1)}{N}=0 \)**

This prime test tells us at once that the Fermat Number-

\[ F = 2^{32} + 1 = 4294967297 \]

is a composite. Here we have-

\[ 4294967297 \mod(6)=5 \]

so that the first part of the prime test is satisfied. Next carrying out a \( f_N \) calculation on \( F=4294967297 \), we find –

\[
f_N:= \text{evalf}(( \text{add}(i,i=\text{divisors(N)})-(N+1))/N) = 0.001560211647
\]

This differs from zero and hence F is composite and not a prime.

Fermat actually claimed that F was prime, but Leonard Euler first showed it wasn’t. Euler spent several months over 200 years ago to actually factor F into the product \( F=(641)(6700417) \). Quite a feat for someone operating without the benefit of electronic computers.
Consider next the Mersenne Number-

\[ M = 2^{61} - 1 = 2305843009213693951 \]

Is it prime or not? First we look at-

\[ 2305843009213693951 \mod(6) = 1 \]

and so see that the number \( M \) is of the type \( 6n+1 \). Next we evaluate –

\[ f_N := \text{evalf}(\frac{\text{add}(i,i=\text{divisors}(N))-(N+1)}{N}) \]

It yields a zero value and hence \( M \) is prime. This fact was first established back in 1883 by Pervushin by a much more complicated and elaborate pre-computer approach.

As a third example, consider the odd number-

\[ N = 74074071 \]

For this case we have-

\[ 74074071 \mod(6) = 3 \]

This number is therefore not of the \( 6n\pm1 \) type and hence it must be composite.

As a final example consider the large number-

\[ N := 342789032178923759 \]

Here the computer commands-

\[ N \mod(6); \quad f[N] := \text{evalf}(\frac{\text{add}(i,i=\text{divisors}(N))}{N}-(N+1)/N); \]

yield-

5 \quad \text{and} \quad f[N] = 0.2355078347

, respectively. That is, \( N \) is a composite number of the form \( 6n-1 \).

The above examples have shown that for a prime to exist it must not only be of the form \( Q = 6n\pm1 \) but also have its number fraction \( f_N \) vanish. The number fraction calculation can become somewhat tedious as \( N \) gets very large.

Graphically we can represent all prime numbers via the following graph-
Note that all primes (marked by red circles) must lie along the diagonal lines $\theta = \pm \pi/3$ and have values $6n \pm 1$. Not every number along these lines will be prime and so the $f_N$ evaluation must still be carried out. The jump in integer values along any of the straight lines is always 6 when going from one turn of the hexagonal spiral to the next. The odd numbers $6n+3 = 9, 15, 21, \text{etc}$ can never be prime. The numbers $6n$, $6n+2$, and $6n-2$ are always even and hence composite (2 excepted).

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