DISTINGUISHING BEWTWEEN PRIME AND COMPOSITE NUMBERS?

It is well known that all positive integers N={1,2,3,4,5,…} fall into two distinct groups. There are the composite numbers-

C={the x=y4,6,8,9,10,12,14,15,16,…}

and the prime numbers-

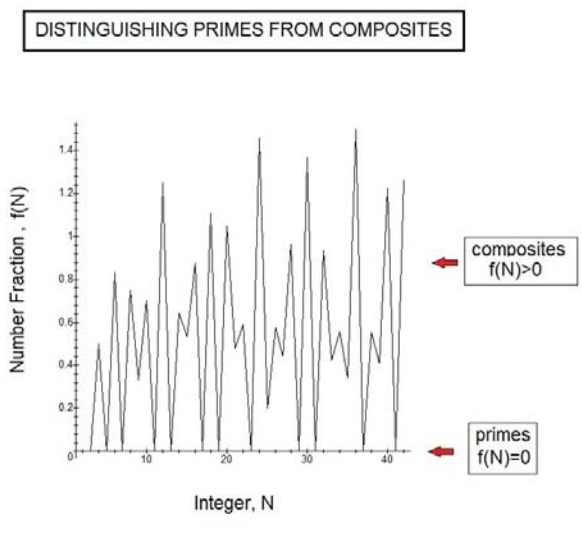
P={2,3,5,7,11,13,17,19,23,…}

With th exception of 2 all prime numbers are odd and must have their Number Fraction f(N) vanish. The number fraction was first defined by us about a decade ago and has the explicit form -

$$f(N) = \frac{[Sum\ of\ \ all\ \ divisors\ of\ \ N] - N - 1}{N}$$

The term in the square bracket of f(N) is just the divisor function of number theory. It is designated by σ(N). All composites have f(N)>0 while all primes have f(N)=0. It is the purpose of this note to expand on these observations and to show how all integers may be plotted as unique points within the x-y plane.

We start with a plot of N versus f(N) for all integers from 2 through 42. The graph looks as follows –



One clearly sees the primes at 2,3,5,7.11.13,17,19,23,29,31,37 and 41. The composites are any of the remaining whole numbers. Some of the f(N)s have values greater than unity. These are characterized by having a large number of divisors. We have termed them super-composites.An example is $N=2^6 \cdot 3^3 \cdot 5^2$=43200 where f(N)=2.6453. Since the values of the sigma point function $\sigma(N)$ is available in most advanced math programs out to at least 30 places , the values of f(N) will also be known to this number of digits. As two examples, let us look at the following 30 digit numbers. The first reads -

N=973172564407356281356210948731

Althougth we may not be able to readily find all its factors, it must be a composite number since f(N)=0.3852722639…. Next looking at-

N=427956710047276592409910356419

, we find f(N)= 0. So it is a prime divisible only by 1 and itself.

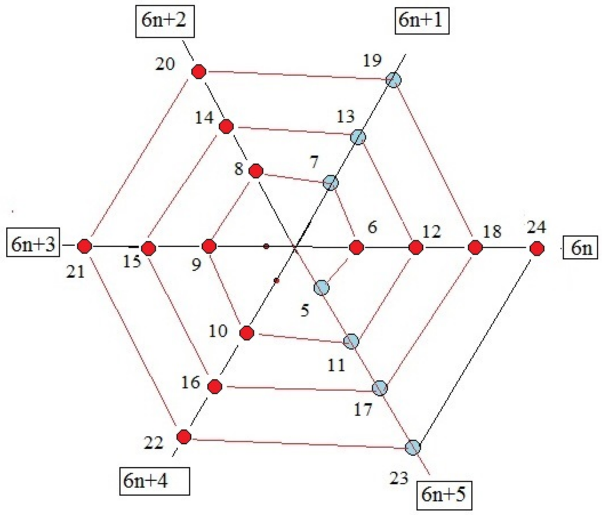
A peculiar thing we first noted about five years ago while plotting f(N) was that all primes P greater than three must have the form $6n\pm1$ without exception. You can clearly see this fact by looking at the N versus f(N) plot above. An alternate way to express this result is to note that P mod(6)=1 or 5. Unfortunately there are also composite numbers C which have this $6n\pm1$ character such as 25=6(4)+1, 77=6(13)-1, and 121=6(20)+1. It restricts our observation to the following statement-

A necessary but not sufficient condition for all primes P greater than three is that they must have the form $6n\pm1$

The f(N)=0 criterion (or its equivalent $\sigma(N)=N+1$) continuous to hold for all primes P but never can be for composites C.

We have found it convenient to express all positive integers as the intersection points between the point hexagonal integral spiral $z=N\exp(i\pi n/3)$ and the six radial lines passing through its vertexes. One has the following picture-

HEXAGONAL INTEGER SPIRAL SHOWING
PRIMES IN BLUE AND COMPOSITES IN RED



The graph starts with N=5 and goes to N=24. Note the primes all fall along either the radial line 6n+1 where N mod(6)=1 or along the line 6n+5 (equivalent to 6n-1) where N mod(6)=5. Note that the spacing between two integers along any of the six radial lines is always equal to a multiple of six.The next prime number lying along 6n+1 is 19+6(3)=31, The number 6(4)+1 -25 remains a composite. We point out that Stanislaw Ulam many years ago wrote down the first few integers in form of a counterclockwise opening spiral on a piece of paper during a rather boring scientific conference The semi-random nature of primes lying along this Ulam Spiral puzzled many mathematicians for years but no one could make sense of it. It was not until our 2008 article(MORPHING ULAM.pdf) that it was first recognized that with an appropriate transformation the Ulam Spiral simplifies to the form given above where all primes lie along just two possible radial lines 6n+1 or 6n-1.

In view of the results summarized in the above graph, we can now find the nearest prime located near any integer N. We first write N-Nmod(6) which brings us to the correct radial line along which the prime P lies. Next we add 6n±1 to define a point anywhere along this line. Expressed mathematically we have-

$$P=N-m+6n\pm1$$

with m=N mod(6). One then applies the search program-

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for n from –b to b do {n,isprime(N-m+6n+1),isprime(N-m+6n-1)}od;
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Here b is an integer very small compared to N.

Let us demonstrate by finding the nearest primes P to-

$N=365789567$ where $m=N \bmod(6)=5$

Carrying out the search using b=3, we find the three primes-

$P=N-m+1=365789563$ $P=N-m-11=365789551$ and $P=N-m+19= 365789581$.

All three lie along the 6n+1 radial line and none along the 6n-1 radial line for the value of b used.

Twin Primes are defined as two primes differing from each other by exactly two. In the above graph we show three of these namely 5-7,11-13,and 17-19. One sees they average to a value of 6n and that one is 6n+1 while the other is 6n-1.They exist whenever the number fraction has the value –

$$f(6n+1)=f(6n-1)=0$$

That is, we have twin primes for n=1,2,3,5,7,10,12,17,18,23,... . It is believed that there are an infinite number of these despite the fact that the spacing between twin primes continually increases with increasing 6n.

Finally let us look at the properties of semi-primes. A semi-prime is written in one of four possible ways-

$$S=(6n+1)(6m+1),\ (6m+1)(6m-1),(6n-1)(6m+1),\text{ and }(6n-1)(6m-1)$$

, with n and m positive integers. When S gets very large, such as for the values used in public key cryptography, the dominant term is 36nm. Also one notices that $S=6r\pm1$, where r is a positive integer. That is, all semi-primes with prime components greater than three also lie along the same two radial lines $6n\pm1$ as the primes do. Let us demonstrate things for-

$$S=21428053=6(3571342)+1$$

, where the prime components are $p=6(679)-1 =4073$ and $q=6(877)-1=5261$. Note that $6nm=6\cdot679\cdot877=3572898$ lies within 0.04% of r=3571342. Semi-primes typically will have their number fraction f(S) lie very close to zero and near 2/sqrt(S). The factoring of very large semi-primes is a difficult and time consuming task. It is the reason why up to the present time the use of public keys in cryptography is still relatively secure.

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