PROBABILITY AND GAMES OF CHANCE

Probability is a measure of the likelihood that an event will occur. Its value will always lie in the range $0 \le p \le 1$. A value of p=1 implies a 100% certainty such as death and taxes. A coin flip has p=0.5 and winning the powerball lottery has a probability of p=0.000000001. Knowledge of probabilities plays a crucial role in understanding games of chance such as card and dice games as well as roulette. It is our purpose here to briefly discuss the probabilities associated with such games of chance.

Early mathematicians such as Fermat, Pascal, Laplace, Bernoulli, and Gauss devoted some of their time to understanding games of chance in terms of the probability of an event occurring. Since that time entire fields of mathematics including probability and statistics have emerged from these early investigations.

FLIPPING OF COINS:

The simplest of the games of chance involves the flipping of coins. In its most elementary form one flips a single coin with an opponent calling heads (H) or tails (T). Since a single coin offers only two possibilities one says that the chance of hitting H or T is 50% each so one has the probability p=1/2 of being right and p=1/2 of being wrong.This is what opposing football teams do at the beginning of a game in order to determine which team will start and which direction toward the goal the winner prefers.

If one flips two identical coins at the same time and records the results there will be four possibilities. These are-

HH HT TH TT

This means there is a p=1/4 probability of getting either two heads or two tails in a row To get one of each occurs two time out of four for a probability of p=0.5. One can say that the probability of getting two heads is p=1/4 and that of not getting two heads is p=1-1/4=3/4. Note that the probability of an event occurring is p and the event not occurring is (1-p).This leads to the obvious conclusion that the probability-

[for an event occurring] = 1-[event not occurring]

Since one does not generally distinguish between the term HT and TH , we may also write things as-

HH 2HT TT

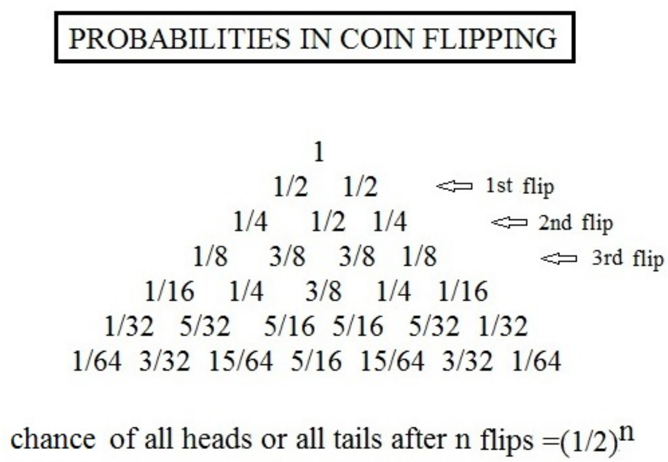
Continue the flipping this time using three identical coins. It produces the eight possible outcomes-

HHH HHT HTH THH TTH HTT THT TTT

or the equivalent-

HHH 3HHT 3TTH TTT

In terms of probabilities we find p=1/8 for three heads or three tails, p=3/8 for two heads and one tail, and p=3/8 for two tails and one head. Extending things to flipping n coins simultaneously one finds the probability pattern



This result applies equally well to taking a single coin and flipping it n times. What is clear from this Pascal like triangle is that to get all heads or all tails after n flips yields the low probability of $p=(1/2)^n$. We can also read off that the probability of an equal number of heads and tails after six flips is p=5/16. This is twenty times more likely than ending up with six heads. Adding up the probabilities along any given row always equals 1.

Flipping a coin can be thought of as a statistically random process were a head or tail comes up equally likely. There are many other examples of such random behaviour with two possible outcomes. One such possibility occurs when looking at the even and odd character of subsequent digits in a random number such as the square root of two. There we have the first 100 digits arranged as twenty groups of five digits each looking like this-

sqrt(2)= 1.4142 13562 37309 50488 01688 72420 96980 78569 67187

53769 48073 17667 97379 90732 47846 21070 38850 38753 43276

41573

If we now take the even numbers as 0-2-4-6-8 and the odd numbers as 1-3-5-7-9, we should expect that the total number of even digits should match the total number of odd digits since for a random process the probability for one or the other is exactly p=1/2. Carrying out the counting we find 26 even and 24 odd numbers in the first 50 digits and 48 even and 52 odd for the first 100 digits. So both cases are close to the fifty-fifty split expected between even and odd numbers. The measured difference from the expected is here 2/50=4/100=4% for both cases.

DICE GAME PROBABILITIES

After the use of coins, the next simplest random number generator used in games of chance involve dice. Known since ancient times, a die has six surfaces on which are embedded the numbers one through six in the form of dots.The numbers are arranged such that the sums for any face and its opposite face always add up to seven. On rolling a die the chance of any of the numbers coming up is one in six yielding a probability of p=1/6. This means to get two ones, twos, etc in a row by throwing the die twice has a probability of (1/6)(1/6)=1/36. This result represents a special case of the **Law of Probabilities** which says the effective probability for n independent events with probabilities of p_1, p_2,p_3,..equals their product. That is-

$$p_{Effective} = \prod_{k=1}^{n} p_k$$

What is interesting to note is that rolling a single die yields on equal probability for any of the numbers 1 through 6 to come up but rolling two or more dice simultaneously changes these probabilities. Let us demonstrate this point by looking at what happens when two dice are involved. There we note there are eleven different face values V={2,3,4,5,6,7,8,9,10,11,12} which come up with different frequencies depending on the number of combinations possible for a given V. Thus snake eyes(2=1+1) , for which there is only one configuration, will show up six times less frequently than a seven which can be constructed by six different combinations. To show this change in probabilities with face value, we have constructed the following table-

FACE VALUE PROBABILITY USING TWO DICE

Face Value V:	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Possible Combinations:	(1,1)	(1,2) (2,1)	(2,2) (1,3) (3,1)	(1,4) (4,1) (2,3) (3,2)	(1,5) (5,1) (4,2) (2,4) (3,3)	(1,6) (6,1) (2,5) (5,2) (3,4) (4,3)	(2,6) (6,2) (3,5) (5,3) (4,4)	(3,6) (6,3) (5,4) (4,5)	(4,6) (6,4) (5,5)	(6,5) (5,6)	(6,6)
Probability for V:	**1/36**	**1/18**	**1/12**	**1/9**	**5/36**	**1/6**	**5/36**	**1/9**	**1/12**	**1/16**	**1/36**

There are a total of (1/6)(1/6)=1/36 different combinations of which there is just one for V=2 but four for V=5 or 9. Thus the probability of throwing snake-eyes(1,1) or boxcars(6,6) is 1 in 36 while throwing a seven increases the probability to 1/6th.

It is important to realize that the probability values given in this table apply only for an infinite number of flips (Bernoulli's Law of Large Numbers) and things will generally differ somewhat from this distribution when a finite number of flips are involved, To show this fact we have rolled a pair of dice 630 times to come up with the following probabilities in percent for each of the different Vs-

V	2	3	4	5	6	7	8	9	10	11	12
630 rolls	4.4	4.1	8.9	12.2	13.2	13.5	11.9	12.1	8.3	7.8	3.8
Infinite rolls	2.8	5.6	8.3	11.1	13,9	16.7	13.9	11.1	8.3	5.6	2.8

Comparing the probabilities for the appearance of a given V between the 630 rolls and the theoretical value for an infinite number of rolls clearly show the same trend but still there are definite departures noted. Gambling casinos usually base their house take on the infinite dice roll values as this is close enough to make gambling a very profitable business for them.

A favorite dice game present in gambling casinos is the game of craps. It requires a thorough knowledge of the above table in order to minimize the houses take. The basic crap game has a table layout as shown-

TOP VIEW OF A CRAP TABLE

DEALER BOXMAN DEALER

PLAYERS

PLAYERS

PLAYERS STICKMAN PLAYERS

In a typical game there are two dealers, the overseer referred to as the boxman , and a stickman for moving the dice and bets around the table. Players stand as indicated in the picture. The basic rules for craps are straight forward. The game has two phases known as the COME-OUT and the POINT .The game begins by a player (known as the shooter) placing a bet of house minimum or more value on either the PASS LINE or the NO-PASS LINE and then throwing the dice toward the opposite wall. Other players can simultaneously enter the betting. If the player has his bet on the PASS LINE and the dice come up with 7 or 11 he wins and doubles his money (chance of this happening equals p=(1/6)+(1/18)=(2/9)=22.2%). If he throws a 2, 3 or 12 he craps out and losses his entire bet. The chance of this happening is (1/36)+(2/36)+(1/36)=1/9=11.1%. If none of the numbers 1, 2, 7, 11, or 12 show up then one enters the second phase by setting a POINT which will be one of the remaining numbers(3,4,5,6,8,9,10) shown by the dice. Say it is 8. Then the shooter rolls the dice again and again until the 8 comes up again to win. He losses if a 7 comes up before hitting the 8 again.

Alternatively a shooter can place his beginning bet on the DON'T-PASS-LINE. Now everything reverses . He wins with a 2, 3 while 12 is kept neutral but looses with a 7 or 11. If none of the numbers 2,3,7,11,or 12 are present on the first die cast , a POINT is established and the game proceeds as before.

There are many variations and other bets one can make in craps. These include betting on individual or combinations of numbers occurring after a single roll of the dice. The house take is generally higher with such variations and hence should be avoided over the PASS-LINE and DON'T-PATH-LINE bets where the house take can be as low as 1.4%.

ROULETTE:

Roulette (French for little wheel) is another favorite game of chance favored by gambling casinos. It consists essentially of a stationary outer wheel with inclined surface onto which a small ball is thrown tangentially. With time it slowly moves inward until caught by one of the pockets on the rotating but slowing inner wheel. Roulette wheels in America haves 18 red numbers and 18 black numbers and pockets plus two green numbers and pockets (marked 0 or 00) placed at equal spacings around the periphery of the wheel as shown-

STANDARD AMERICAN ROULETTE WHEEL WITH 38 POCKETS
(two house pockets schown in green are marked by 0 and 00)

House Advantage Equals 2/38=5.263%

The wheel itself sits on a felt covered table which contains information about the types of bets a player makes before the wheel is spun, The simplest bets to make are either black-red or even-odd. The odds here of winning are p=(18/38)=47.4% for one spin of the wheel. If the ball ends up in either green pocket the player looses. The house advantage in American Roulette is 2/38=5.3%. This is a quite high house advantage which will with repeated tries eventually lead to the players ruin with the house ending up holding all the money. The Roulette Wheels in Europe have only one green pocket with a total of 37 numbers. This type of roulette cuts the house take down to 1/37=2.7% and it is therefore better to play Roulette in Monte Carlo than Las Vegas. Since I am not a gambler, I will generally place a limit of less than $50 when playing Roulette at a casino. My experience at Manaco during several visits there during the last few decades has shown me the wisdom of such a restriction. Every once in a while an amateur player will come up with an old idea known as the Martingale. Here one starts with a bet of say $5 on even-odd. If he wins he takes the $10 and goes home. If he looses he places a second bet this time for $10. If he wins he pockets the $20 and leaves with a net $5(=20-10-5) win. If he looses he bets again this time with $20. Continuing this process on the gambler either collects his winning or doubles the next bet. At first glace this looks like a sure fire way to beat the house until it is realized that $(\$5)^n$ soon becomes a very large number either hitting the table limit or the limited resources of the player. Either way large losses will occur when the next bet is not allowed . Playing a martingale for only a few bets might lead to a small gain but at most this will amount to the size of the original bet.

Roulette also lends itself to all types of different bets including betting on a group of numbers or even one individual number. In most cases the house take rises for such variations so it is advisable to not try such bets.

PRICE FLUCTUATIONS IN STOCK MARKETS:

Another important area where probability plays a role is in stock market price behavior. Although most investors believe prices have associated with them a trend driven by chart patterns(Chartists) or earnings trends(Fundamentalists), convincing arguments can be made that short term price movements are random and yield a non-predictive behavior not unlike that of coin flipping. One of the major proponents of this random behavior model is B. Malkiel in his book “A Random Walk Down Wall Street”. He suggests that short term price fluctuations have no predictive value other than to suggest how far the price has departed from a running average. This leads to the obvious conclusion that stock markets for short term traders act essentially as gambling casinos with brokerage commissions playing the role of the house take. Like Warren Buffett, I believe that stock price long term behavior show certain trends such as was clearly visible with stocks like GOOGLE, MICROSOFT, AMAZON, PANERA before their historic price rises. Looking at price charts of these stocks clearly show short term fluctuations which are essentially unpredictable other than to indicate when things are momentarily overvalued or undervalued. The best investment strategy for the longer term investor seems to be to buy high quality stocks and hold while ignoring random daily and weekly fluctuations. For conservative investors buying a S&P500 exchange traded fund (ETF) and adding to it periodically is probably the optimum strategy.