PROPERTIES OF Q PRIMES

Several years ago we found by a simple-prime number count that all primes five or greater have a necessary but not sufficient condition that they have the form $6n \pm 1$. Neglecting those forms of $6n \pm 1$ which are composite such as 25, 35, 49, 65, etc., we are left with the following prime number sequences:

$6n+1 = \{7, 13, 19, 31, 37, 43, 61, 67, 73, 79, 97, 103, 109, 127, 139, 151, 157, 163, 181, 193, 199, 211, 223, 229, 241, 271, 277, 283, 307, 313, 331, 337, 349, 367, 373, 379, 397, 409, 421, 433, 439, 457, 463, 487, 499, 523, 541, 547, ... \}$

and-


We have chosen to call these primes the Q Primes. The obvious properties are that

(1)-The difference between any of the primes, in either the $6n+1$ or $6n-1$ sequences, differ by factors of six. Thus 137-29=6(18) and 503-239=6(44).

(2)-Twin primes $T[p,q]$ (also known as double primes) must have the form $p=6n+1$ and $q=6n-1$ so that $p+q=12n$ and $p-q=2$. Here $pq=T=36n^2-1$. An example of a twin prime is $T=899$ where $n=5$, $p=31$ and $q=29$.

(3)-All semi-primes $N=pq$ must have the form $N=36nm+6(n+m)+1$ or $N=36nm+6(n-m)-1$ and the property that $N=6k \pm 1$. An example is $N=77=6(13)-1$ where $p=6(1)+1=7$ and $q=6(2)-1=11$.

In thinking about the obvious relation of Q Primes and the forms $6n+1$ or $6n-1$, it became clear to us several years ago that one could conveniently plot these primes as points along the radial lines $6n+1$ and $6n-1$ where they cross a hexagonal integer spiral defined in the complex plane by the vertex points—

$$N=r \exp(i\pi n/3) = |N| [\cos(\pi n/3) + i \sin(\pi n/3)]$$

where $r$ is the magnitude of any integer $N$ and $n$ represents any positive integer. Connecting neighboring vertexes with straight lines produces what I call the Hexagonal Integer Spiral. Here is what it looks like when we superimpose the six radial lines $6n$, $6n+1$, $6n+2$, $6n+3$, $6n+4$, and $6n+5$.
Note that all positive integers are located at the intersection of a spiral vertex and one of the six radial lines. Marking the Q Primes on this spiral in light blue circles produces the mathematically pleasing allignment shown-
For years mathematicians and computer specialists have been trying to make sense out of the semi-regular distribution of primes found in a related spiral known as the Ulam Spiral hoping this will produce new information on primes. Apparently they have not realized that all they are seeing but not recognizing is that primes above three are odd numbers and have the form $6n \pm 1$. Go to

http://www2.mae.ufl.edu/~uhk/MORPHING-ULAM.pdf

for a discussion of this point.

We can use modular arithmetic to show that the product of the Q primes $p=6n+1$ and $q=6m+1$ yields the odd number $N=36nm+6(n+m)+1$. Provided that the primes $p$ and $q$ are both greater than three, we have $p \mod(6)=1$ or $-1$ and $q \mod(6)=1$ or $-1$. Hence $N=pq$ must satisfy $N \mod(6)=1$ or $-1$. This means any semi-prime $N=pq$ must also lie along either the radial line $6n+1$ or the radial line $6n-1$ without exception. Take $p=41$ where $p \mod(6)=-1$ and $q=103$ with $q \mod(6)=1$. We have that $N=41(103)=4223$ is an odd number with $N \mod(6)=-1$ (which is the same as saying $+5$)

The sum of any two Q primes $p$ and $q$ equals-

$$S = p+q=2[1+3(n+m)]$$
This sum is always an even number. For example, if \( p=337 \) where \( 337 \mod(6)=1 \) and \( q=823 \) where \( 823 \mod(6)=1 \), we have \( p+q=1160 \). So, we have \( 1160 \mod(6)=2 \). Thus the sum \( S=1160 \) is an even number lying along the radial line \( 6n+2 \).

We can conclude from these modular arithmetic observations that the sum of two \( Q \) primes is always equal to an even number. The reverse of this statement is the famous Goldbach Conjecture that any even number can always be expressed as the sum of two primes.

Adding together \( N \) \( Q \) primes we find:

\[
S = \sum_{n=1}^{N} Q_n
\]

is an even number when \( N \) is even and an odd number when \( N \) is odd. So the four \( Q \) prime sum-

\[
S=71+467+211+547=1296
\]

is an even number. Also since \( S \mod(6)=0 \) we know that 1296 lies along the horizontal line \( 6n \) in the above graph.

We can also work out the partial sums of both the \( 6n+1 \) and \( 6n-1 \) \( Q \) primes. We have-

\[
P_{6n+1} = \{7,20,39,70,107,150,211,278,351, ... \}
\]

and-

\[
P_{6n-1} = \{5,16,33,56,85,126,173,226,285, ... \}
\]

The terms in these partial sum sequences show no regular pattern other than that they increase in size monotonically and neighbors are odd-even. The latter follows from the earlier result on the sums of \( Q \) primes.

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