-   DETERMINING SHADOW TRAJECTORIES

It is well known that a vertical pole (Pelekinon) set on a horizontal surface will project a shadow on the ground which varies with the local latitude(LAT), the sun's declination(DEC) and the hour-angle(HA). At local noon , where the hour angle HA=0, the celestial spherical triangle predicts that the sun's altitude(ALT) is given by-

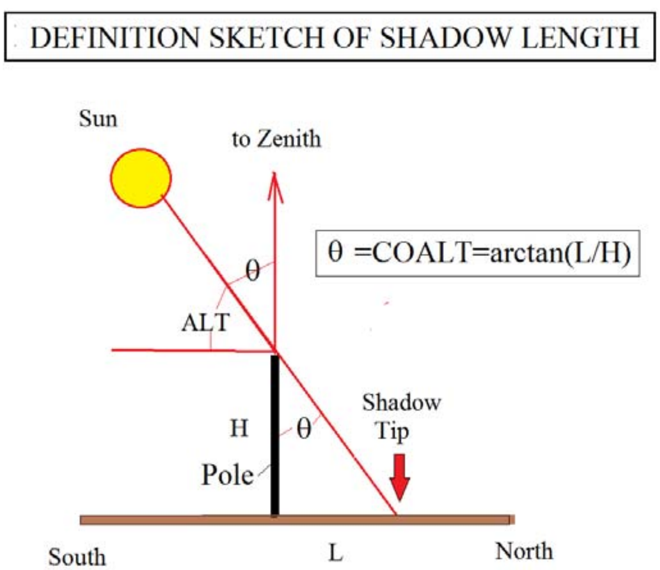
$$ALT=90deg-(LAT-DEC)$$

So at local noon the sun will be at an altitude of ALT=90-30-23.5= 36.5deg during the Winter Solstice for someone located at LAT=30N degrees. Anyone living above the arctic circle at LAT≥90-23.5=66.5degN will not see the noon sun at all on that day. Clearly if one follows the value of ALT throughout the year it will vary in a continuous manner over the range-

$$(67.5-LAT)<ALT<(113.5-LAT)$$

Here in Gainesvile, Florida, where the latitude is LAT=29.6 deg N , the sun's altitude will vary between ALT=37.9degree at the Winter Solstice to ALT=83.9 deg at the Summer Solstice.

A schematic of the expected local noon shadow length L produced by a vertical pole of height H follows-



We see here that the shadow length at local noon is-

L=H tan (COALT)=H tan(LAT-DEC)

It is known that the sun's declination varies throughout the year in an approximately sinusoidal manner. The declination is found to be close to-

$$DEC = 23.5\sin(\frac{2\pi\, n}{365})$$

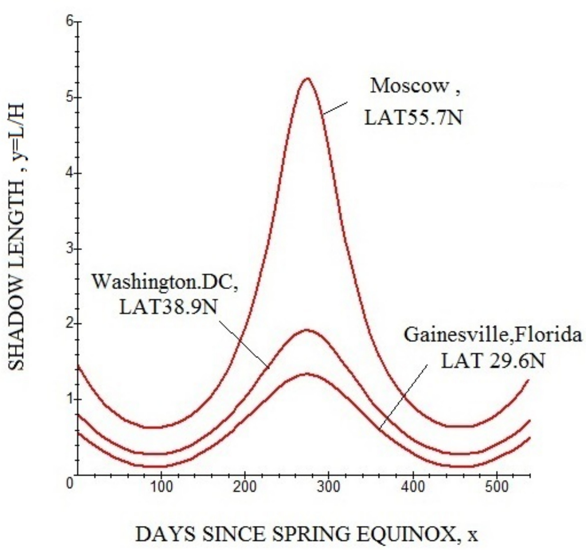
, where n represents the number of days after the Spring Equinox. When DEC=+23.5 deg the sun is above the Tropic of Cancer while DEC=-23.5 means the sun is above the Tropic of Capricorn. Substituting this value of DEC into our L equation produces the result-

$$y = \frac{L}{R} = \tan\left[LAT - 23.5\sin(\frac{2\pi\, x}{365}) \right]$$

Here we have set y=L/R and x=n. One can plot this last equation for any latitude LAT in the Northern Hemisphere to produce a unique shadow tip trajectory at local noon. Let us choose the following three locations-

Gainesville, FL	LAT= 29.6N	LONG= 82.3W
Washington, DC	LAT= 38.9N	LONG= 77.0W
Moscow, Russia	LAT= 55.7N	LONG= 37.6E

The shadow curves corresponding to these locations look like this-



The shadow length reaches its maximum value at the Winter Solstice and its minimum at the Summer Solstice. The closer one gets to the equator the lower the maximum of a given curve becomes. In Moscow a vertical post of H=1 meter will cast about a 5.4 meter long shadow on a horizontal surface on December 21. Also the sun at local noon on that date will be only ALT=10.8 degrees above the horizon. So winter is probably a pretty dreary time for Moscow citizens and even more so for Swedes in Stockholm and Norwegians in Oslo. Here in Gainesville we don't have the problem of not enough sun, however, we pay for it through oppressive heat and humidity during the summer months of June through September.

Note that one can determine the circumference of the earth by noting the length of the noon shadow at two different latitudes along the same longitude. This is essentially what Eratosthenes of the Greek School in Alexandria did some 2200 years ago. Let us repeat his experiment here in reverse. We have Gainesville, FL at LAT29.6N and LONG82.3W and Columbus, Ohio at LAT40N and LONG83W. They have about the same longitude. At the Winter Solstice the values for shadow lengths are $y_{GNV}=1.3319$ and $y_{COL}=2.0056$. So taking the arctan of both ys, and then subtracting the results from each other produces $\Delta\theta=0.1803$ radians. Now looking at the earth as a sphere of Radius R=3960 miles, we conclude the distance between Gainesville Florida and Columbus, Ohio equals approximately-

$$D=R\{\arctan(2.0056)-\arctan(1.3319)\}=714 \text{ miles}$$

The actual distance to drive between these cities is 877 miles since the roads connecting them do not form a great circle route. The exact great circle distance can be calculated with aid of spherical geometry. It is given by the formula-

$$\cos(D) = \{\cos(COLAT_{GNV})\cos(COALT_{COL}) - \sin(COLAT_{GNV})\sin(COLAT_{COL})\cos(\Delta LONG)\}$$

Here COLAT= 90deg-LAT and $\Delta LONG = LONG_{GNV} - LONG_{COL}$. Substituting the above coordinates into this equation produces-

$$D=\arccos\{\cos(60.4)\cos(50)-\sin(60.4)\sin(50)\cos(82.3-83.0)\}=0.18178$$

Multiplying this by $60(180/\pi)$ yields nautical miles and then by another 1.1515 yields-

$$D= 0.18178*60*180*1.1515/\pi=719.63 \text{ miles}$$

This is within six miles of our earlier cruder approximation and some 157 miles less than what a road trip will require. An airplane has the advantage over road or rail travel in that it is most of the time free to fly a great circle route between any two points on earth.