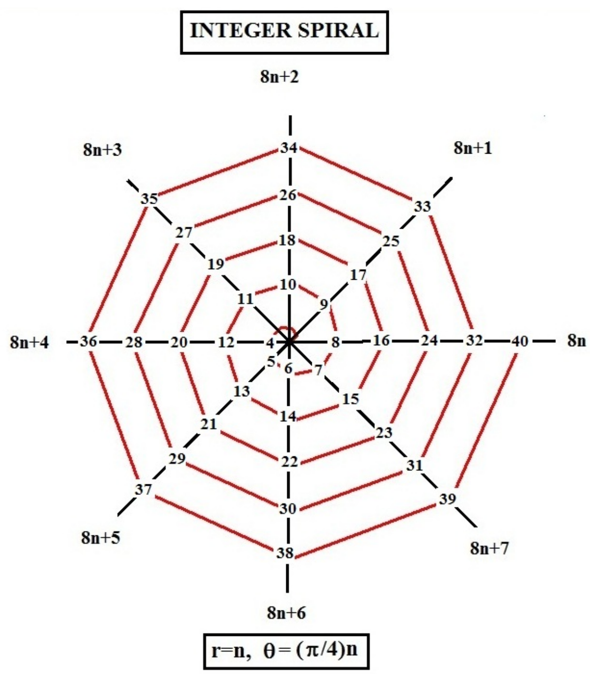
RELATIONSHIP BETWEEN INTEGER SPIRALS AND SPIDER WEBS

In the last several years we have-examined certain unwinding spirals which are intersected by radial lines passing through the spiral origin. We have found that the intersection of such spirals and the superimposed radial lines allows for a perfect sequential representation of all positive integers N. These integers are located at the curve intersection points expressed in polar coordinates as-

$$[r,\theta]=[N,2\pi N/n]$$

, where n represents the number of equally spaced radial lines. A graph of one of these integer curves corresponding to n=8 produces the following figure after eight radial lines are added-



For this integral spiral the numbers are located as indicated starting with r=0 at θ=0. One can quickly determine at what intersection any other positive integer N lies by carrying out a mod operation. So, for example, the integer N=31 has N mod(8)=7 and so lies on the 8n+7 radial line at the 24/8=3rd turn of the spiral. Notice here that one turn of the spiral equals a change of eight units. We are using straight line connections along the spiral between neighboring integers. It is possible to use other connections where the connecting curves along the spiral have a non-linear nature. An example would be using the Archimedes Spiral $r=4\theta/\pi$ where the spiral is continuous at the intersection points with the radial lines.

What strikes one about this form of integer representation , which represents an improved version of the earlier Ulam spiral, is, among other things, the similarity to a spider web as found in nature. On scanning the internet we found many excellent photos of such spider webs. One of these is the following-

SPIDERWEB AS SEEN IN NATURE

(source-photography. nationalgeographics.com)

If one looks at this spider web from the back-side, one sees an unwinding counter-clockwise spiral intersected by 14 radial lines. The spiral segment between any two neighboring radial lines are not quite straight. Rather they have a scallop structure although some are nearly straight. In its construction the spider first produces the radial silk strands anchoring them to outer support strands and merges them to a common point at the origin. Next the spider moves along a spiral trajectory laying a continuous strand to complete the web. The strands have been found to be stronger than steel and are coated with a sticky substance which allows for the trapping of insects. The spider itself sits near the origin of the spun web and waits for the arrival of his prey. Like most of nature's constructs, a spider web shows departures from one having perfect radial and angular symmetry. This fact does not, however, detract from the fact that they work quite effectively.

We are able to simulate the above shown spider web with a model consisting of 14 radial lines and a spiral consisting of connected straight lines between neighboring radial lines. The computer command for generating such an ideal spider web pattern is given by-

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listplot([seq([N,Pi*N/7],N=0..60)],coords=polar,color=red,thickness=2,scaling=constrained,axes=none);
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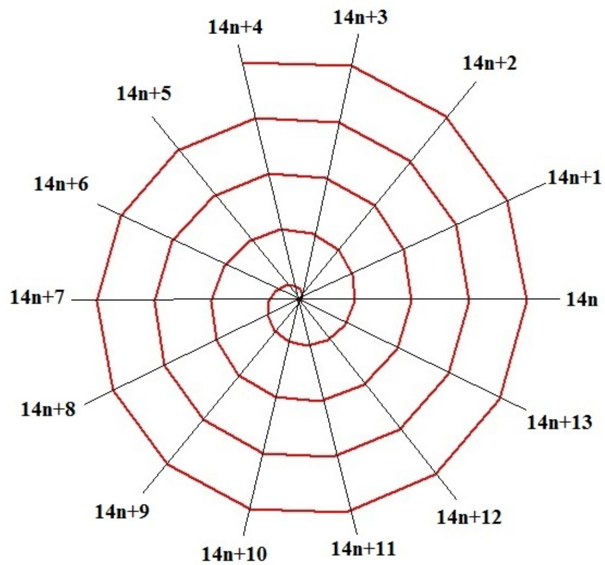
Once this plot has been obtained, one draws in the 14 radial lines, to produce the analytically generated spider web shown-

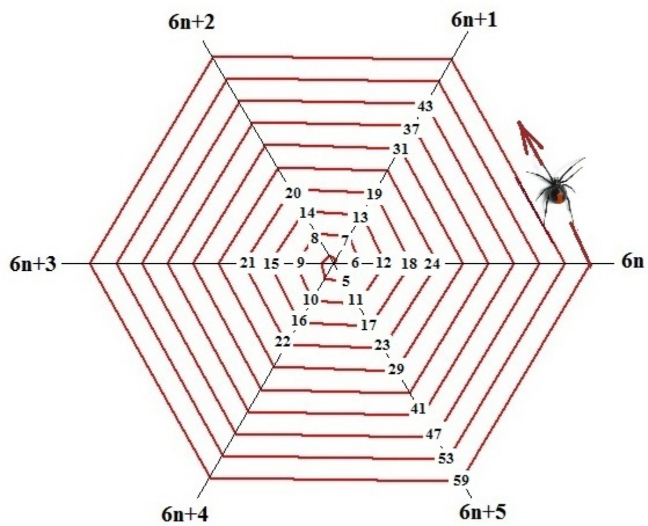
SIMULATED SPIDER WEB WITH 14 RADIAL LINES



One could envision a structure like this , when elongated in the z direction, as an effective electrostatic filter for the trapping of dust particles in certain combustion processes. Such a scroll filter would be much easier to construct and clean than a standard rectangular grid filter.

There is of course no real restriction on the number of radial lines employed in an integer spiral. A very interesting integer spiral which we first found two years ago is one where the intersection points between the spiral and the radial lines are located at the polar coordinates $[r,\theta]=[N,\pi N/3]$. This integer spiral is intersected by six, equally spaced, radial lines and produces the figure shown-

Spider Net and All Positive Integers



Q Primes lie along 6n+1 and 6n+5 Lines Only

Our interest in this particular integer spiral was motivated by the fact that all Q primes(ie-those primes with p=5 or greater) have the form 6n±1. This means that all primes with the exception of 2 and 3 will lie along the two radial lines 6n+1 and 6n+5 as shown in the graph. I have added the working spider to the graph just to indicate why I sometimes call this type of integer spiral a spider web. Note that the numbers lying along the remaining four radial lines must be composites. I have not typed in all these composite values in order to save time. There is a six integer jump for each turn of the spiral. Thus the next composite number after 20 along the 6n+2 radial line is 26.

To locate a number N on this n=6 integer spiral, one must first perform an N mod(6) operation to determine along which radial line the number lies. Then if the mod operation yields 0, 2, 3, or 4, the number N is composite. If it yields 1 or 5 (which is equivalent to -1) it may or may not be a prime number. To determine if it is prime requires a prime test such as having the number fraction f(N)=0. The number fraction is defined as f(N)=[σ(N)-(N+1)]/N, where σ(N) represents the sum of all divisors of N. Here are a few examples:

N=4637892156081 has N mod(6)=3 and so is a composite

N=3135186363329 has N mod(6)=5 and so may or may not be composite. Doing a number fraction test shows f(N)=0 and so N is a Q prime.

N= 345861092371 has N mod(6)=1 and so may or may not be composite. Doing a number fraction test shows f(N)=0.002638523523 and thus N is a composite.

U.H.Kurzweg
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