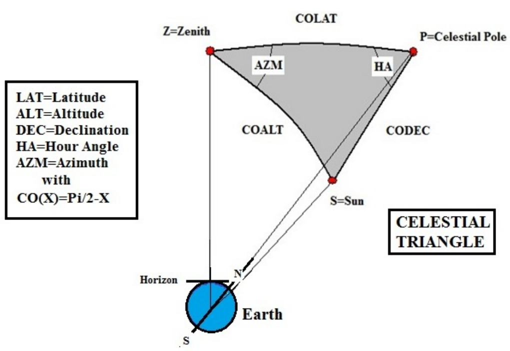
SUNRISE, SUNSET, AND LOCAL NOON THROUGHOUT THE YEAR

These days most individuals don't have a good idea of why the sun rises and sets at different times throughout the year and why this varies with one's latitude and longitude and the sun's declination relative to the equator. Most will just rely on their local TV weather forecast , the internet, newspapers or the almanac to get this information but don't really understand how these numbers come about. We remedy this situation here by deriving these quantities from scratch.

Our starting point is the celestial triangle as shown-



This is a spherical triangle drawn onto the celestial sphere having the zenith (Point directly above the observation point), the North Star (Celestial Pole), and the Sun as its three vertices. The angle at the Zenith is the azimuth AZM and the angle at the North Pole is the Hour Angle HA. The sides of this spherical triangle are the COLAT between the Zenith and North Pole, the COALT between the Zenith and Sun, and the CODEC between the Sun and the North Pole. Here ALT is the altitude of the Sun above the horizon making COALT=π/2-ALT. Likewise the latitude LAT=π/2-COLAT. Now from any math handbook we have that the Law of Sines for the Celestial Triangle is-

$$\frac{\sin(AZM)}{\sin(CODEC)}=\frac{\sin(HA)}{\sin(COALT)}$$

and the Law of Cosines reads-

$$\cos(COALT) = \cos(CODEC)\cos(COLAT) + \sin(CODEC)\sin(COLAT)\cos(HA)$$

At sunrise and sunset ALT=0 while at local noon the hour angle is HA=0. Doing first the local noon part we get –

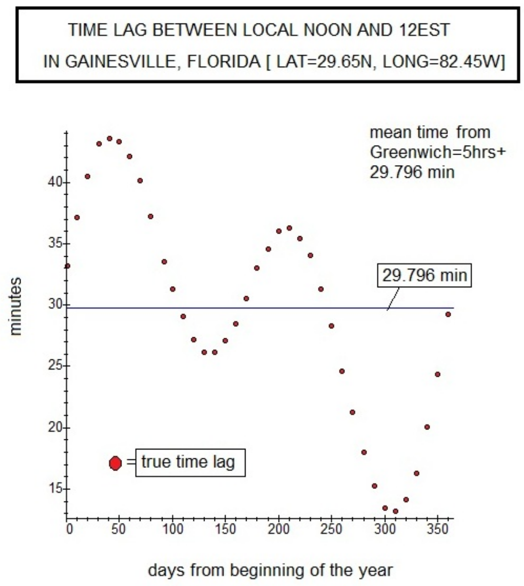
$$\cos(COALT)=\cos(COLAT-CODEC)$$

which means that-

$$ALT=(\pi/2)-\arccos[\cos(COLAT-CODEC)]$$

This represents the altitude of the Sun above the horizon at local noon. Here in Gainesville, Florida our position is LAT=29.65 N and LONG=82.45 W. During the Winter Solstice on Dec.21 one has DEC=-23.5 leaving us with a noon altitude of 53.15deg . During the Summer Solstice on June 21 we get a noon altitude of 84.85deg. That is, during the Winter Solstice the sun reaches to within 90-53.1=36.9deg from the Zenith while during the Summer Solstice it lies only 5.15deg from the vertical at local noon.

It must be remembered that local noon here is not 12 noon EST since we are located an uneven number of hours west of Greenwich, England. Our time relative to Greenwich is 82.45/15=5.4966hrs later, so we are closer to 12:30 Eastern Standard Time for our local noon. Two days ago (September 17, 2018) I was measuring when the true noon occurs here in Gainesville by noticing the shadow cast on the tile floor by some partially open venetian blinds in our washroom. The sun was directly south at 1.24 pm. The extra hour is due to the fact that we are still at the moment on Daylight Savings Time. In a few weeks we will return to regular Eastern Standard Time by falling behind one hour. The extra remaining few minutes departures from 12.4966 occurring throughout the year has to do with the Earth's slightly elliptical path around the sun. These minute departures as they actually occur in Gainesville (LONG=82.45degW) is summarized in the following graph-



From the graph we see that local noon can be as little as 12 minutes after 12EST and as much as 44 minutes after 12EST here in Gainesville. The exact Sun's declination DEC can be found in any nautical almanac. The declination is given approximately by the sinusoid-

$$DEC=23.5\sin(2\pi n/365)$$

,where n measures the days after the Spring Equinox on March 21. So during the Summer Solstice on June 21 we predict the correct value of DEC=23.5deg . At the Winter Solstice we get DEC=-23.5deg.

We next want to look at the time of sunrise and sunset throughout the year. At both sunrise and sunset the Sun's altitude is zero so that COALT=$\pi/2$ rad. So from the above Law of Cosines we have-

$$\cos(HA) = -\{\cot(CODEC)\cot(COLAT)\}$$

So if we know the declination of the Sun and the latitude of the observer, we can get the hour angle HA away from local noon. At the Summer Solstice here in Gainesville we expect the sun to rise at HA=$\cos^{-1}$\{-\{cot(90-23.5deg)cot(90-29.65)deg\} = 104.33deg=6.955hrs before local noon at 1.466 pm daylight Savings Time and to set at 6.955hrs after 1.466pm. That is sunrise at 6.511am and sunset at 8.421pm Daylight Savings Time. We are expressing time in fractions of an hour. For the Winter Solstice we get HA=75.669deg=5.0446hrs. Local noon on that date falls at 12.466pm EST. So the Sun rises at 7.422am and sets at 5.511pm. The use of military time where 2.567 pm equals 14.567 is useful in the present calculations. Today (Sept.19, 2018) the almanac shows DEC=1.86degN. So we find HA=91.058deg=6.0706hrs and local noon occurs at

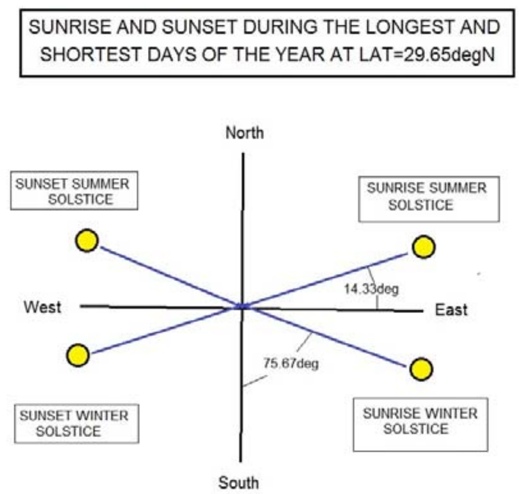
13.235 military time. Hence we have the sunrise in Gainesville at 13.235-6.0706=7.164amDST and sunset at 13.235+6.0706=7.3056pmDST. You can verify these times by going to the Google search engine and typing in sunrise- sunset- Gainesville-Florida . The larger the latitude of an observation point becomes the longer the days in the summer and the shorter the days in the winter. Above the arctic circle at 90-23.5=66.5deg LAT the Sun never rises above the horizon at the Winter Solstice.

We can calculate the time between sunrise and sunset at any latitude in the Northern Hemisphere by the formula-

$$\text{Daylight in Hours=DH=2HA=}2\cos^{-1}\{-\cot(CODEC)\cot(COLAT)\}$$

So at Summer Solstice DH=13.938 hours at LAT=30deg and DH=16.672 hours at the higher latitude of LAT=60deg.

We can take some of the above results and draw an interesting compass graph showing sunrise and sunset during the longest and shortest day of the year. Here is the graph as it applies to Gainesville, Florida at LAT=29.65N-



Note that sunrise on June 21 lies on a straight line passing through the origin and intercepting sunset on December21. This symmetry is the reason I suggested several years ago that opposite to the heal stone at Stonehenge, England (used for measuring midsummer sunrise) there should be evidence for a second stone along the same line opposite of Stonehenge for measuring sunset during the Winter Solstice.

Finally I would suggest that one do away with daylight savings time. It is a real inconvenience to have to adjust one's multiple clocks and watches twice a year. My suggestion would be to move clocks ahead by one-half hour from the present standard time and keep things there throughout the year.

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Gainesville, Florida