EXPONENTIAL VECTORS AND SUPER-COMPOSITES

It is well known that any whole number \( N \) can be written as-

\[
N = \prod_{n=1}^{b} (p_n)^{a_n}
\]

, where \( p_n \) is the nth prime and \( a_n \) a specified exponent. So, for example, \( N=3675 \) can be written as-

\[
N = 3675 = 3 \cdot 5^2 \cdot 7^2
\]

with \( a_2=1, \ a_3=2 \) and \( a_4=2 \).

Associated with such an expansion, which is accomplished by our mathematics program by the operation ifactor, we have a unique exponent vector-

\[
V(N) = [a_1 \ a_2 \ a_3 \ldots a_b]
\]

For \( V(3675) \) this vector reads-

\[
V(3675)=[0 \ 1 \ 2 \ 0 \ 0 \ 0]
\]

The zero elements in the vector indicate those primes not present in the expansion. A very interesting property of such exponential vectors is that one can often choose a second vector \( W \) such that \( \sqrt{VW} \) is an integer. This property can be used to advantage in factoring semi-primes. Our purpose in this article is to concentrate on the relation between exponential vectors and a special type of number labeled earlier by us as a super-prime.

We define super-primes as all those numbers where the number fraction \( f(N) \), defined by-

\[
f(N) = \frac{\sigma(N) - N - 1}{N}
\]

, is much greater than unity. Here \( \sigma(N) \) is the divisor function of number theory which represents the sum of all divisors of a number \( N \). For prime numbers \( f(N) \) is always zero while for composites it is greater than zero. When exceeding values above one it is likely to be a super-composite. These super-composites can be recognized graphically by their height relative to their immediate neighbors.
Consider the number $N=3628800$. If we plot $f(N)$ for this number over a range $N \pm 10$ we get the graph:

![Graph showing $f(N)$ for $N=3628800$ over a range $N \pm 10$.]

This graph clearly shows that 3628800 is a super-composite. Note how $f(N)$ is considerably larger than other values in its neighborhood. Also a weak symmetry in the values of the number fraction are noted about $N=3628800$. The exponent vector associated with this super-composite is gotten by first writing out the ifactor:

$$\text{ifactor}(3628800) = 2^8 \cdot 3^4 \cdot 5^2 \cdot 7$$

and then collecting the exponents to yield:

$$V = [8 \ 4 \ 2 \ 1 \ 0 \ 0 \ 0]$$

An examination of this vector shows a particular concentration of element values for the lowest primes. This makes sense since a number such as 2, 3 and 5 taken to higher powers should provide more divisors for the sigma function. For this particular $N$ we find all the divisors to be:

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 24, 25, 27, 28, 30, 32, 35, 36, 40, 42, 45, 48, 50, 54, 56, 60, 63, 64, 70, 72, 75, 80, 81, 84, 90, 96, 100, 105, 112, 120, 126, 128, 135, 140, 144, 150, 160, 162, 180, 12600, 16800, 44800, 11340, 56700, 22400, 28350, 3780, 4480, 5670, 8960, 7560, 18900, 26880, 18144, 2268, 1512, 4536, 3024, 9072, 6048, 1344, 4032, 12096, 36288, 24192, 16128, 48384, 2688, 8064, 5376, 1400, 1575, 2100, 2800, 3150, 11200, 113400, 75600, 226800, 151200, 90720, 453600, 4200, 4725, 5600, 6300, 1680, 8400, 9450, 2240, 2835, 3360, 378,
Adding these divisors together (skipping N and 1) and dividing by N produces the number fraction-

\[ f(3628800) = 3.225663304 \]

clearly falling into the super-composite realm.

Next, considering a second vector whose elements have their largest values associated with the lowest primes. We look at-

\[ V(N) = [4 \ 3 \ 1 \ 1 \ 1 \ 0] \]

This is equivalent to the six digit long number-

\[ N = 166320 \]

A plot of the number fraction \( f(N) \) over its neighborhood \( N \pm 10 \) looks like this-
Clearly we have a super-composite with $f=3.294366282$. This time we also note that the immediate neighbor 166319 is a prime while the other neighbor 166321 equals a semi-prime $59 \times 2819$. Semi-primes tend to have very low values near zero but never exactly zero.

The two specific examples of super-composites discussed above suggest one can generalize things to find multiple additional super-composites. Their exponent vectors will typically have the form:

$$V(N)=[a,b,c,d,e,..] \quad \text{with } a>b>c>d>e>f$$

Among the infinite number of other exponent vectors of this type, we looked at:

$$V=[19 \ 4 \ 2 \ 1 \ 0 \ 0 \ 0] \quad V=[14 \ 6 \ 2 \ 1 \ 0 \ 0] \quad \text{and} \quad V=[6 \ 5 \ 4 \ 3 \ 2 \ 10]$$

They were all found to be super-composites with the respective $f(N)$ values of:

$$f=3.233928943 \quad , \quad f=3.249354932 \quad \text{and} \quad f=3.294366282$$

Note that when dealing with exponent vectors where the elements are more evenly distributed the values for $f$ will drop most likely putting such numbers below the super-composite range. For example $V=[0 \ 2 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1]$ yields $f=0.9709255565$ which makes $N=157905$ just an ordinary composite but not a super-composite.

Super-composites $N$ are sometimes bounded by primes at $N\pm 1$. This occurs for $N=60$ where $f(60)=1.78333$ while $f(59)=f(61)=0$. So here $N=60$ is bounded by a twin prime. As we have shown in an earlier note, twin primes occur under those conditions where both $(6n-1)$ and $(6n+1)$ are simultaneously prime. So $6n$ will always be the value of $N$. 

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**SUPER-COMPOSITE $N=166320$ AND ITS EXPONENT VECTOR $V=[4 \ 3 \ 1 \ 1 \ 0 \ 0 \ 0]$**

![Graph showing the distribution of $f(N)$ values for $N$ ranging from 166310 to 166330. The peak value of $f(N)$ is at $N=166320$ with $f=3.294366282$.](image.png)
sandwiched between a twin prime. This N may or not be a super-composite depending on the value of f(N).

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