HEAT FLOW INTO A TEMPERATURE WELL

About thirty tears ago we came up with a new way to transfer heat at accelerated rates between two fluid reservoirs maintained at different temperatures by oscillating fluid axially within a bundle of open ended capillaries connecting the reservoirs (see Journal of Fluid Mechanics **156**, 291-300,1985). The secret to the enhanced heat transfer found with this technique is that higher frequency oscillations can produce very large transverse heat transfers governed by the Fourier heat conduction law-

$$\dot{q} = -kAgrad(T)$$

Here k is the thermal conductivity of the fluid, A the cross-sectional area, and gradT the temperature gradient. It is our purpose here to look at a variation of this problem in which we ask the question how fast one can transfer heat into a temperature well if the well is periodically reset.

To keep things simple, we consider a one dimensional temperature well in analogy with a quantum well. The temperature in the well is initially set at $T=T_C$ and the well extends from x=-1 to x=+1. Outside this range where |x|>1 we have the higher temperature of T_H . To keep the mathematics to a minimum we assume that both the thermal diffusivity α and the thermal conductivity k are kept the same both inside and outside the well. Recall that-

$$\alpha = k/(\rho c) \quad with \quad \rho = density \quad and \quad c = specific\ heat$$

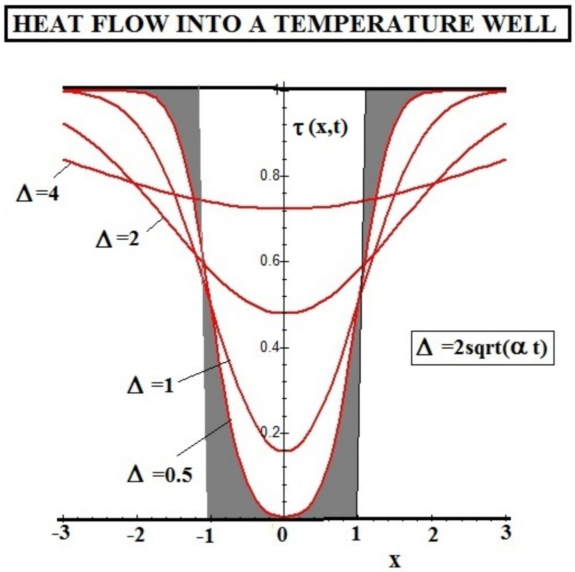
The governing equation for the material is-

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad subject\ to\ T(t, \pm\infty) = T_H\ and \quad T(0,x) = \left\{ \begin{matrix} T_H\ for\ |x|>1 \\ T_C\ for\ |x|<1 \end{matrix} \right\}$$

One can perform a Fourier transform on this equation with respect to x. Doing so and incorporating the initial and boundary conditions yields the closed form solution-

$$\tau(x,t) - \frac{T(x,t)-T_C}{T_H - T_c} = \left\{ 1 - 0.5[erf(\frac{x-1}{\Delta}) + erf(\frac{x+1}{\Delta})] \right\}$$

where $\Delta = 2\sqrt{\alpha t}$ is an indication of the time t and erf is the error function. In the following graph we show the non-dimensional temperature $\tau(x,t)$ for several different values of Δ -



(Shaded regions show heat tranferred at Δ=0.5)

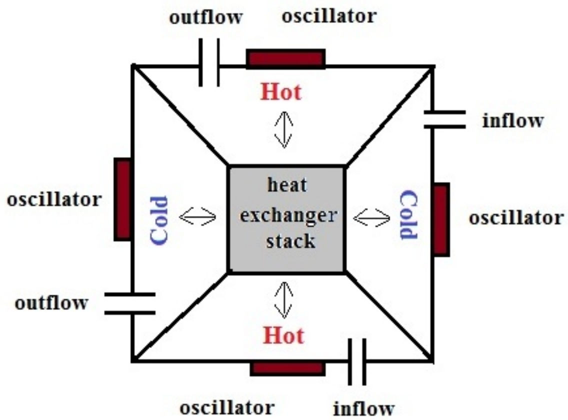
What is noticed is that the temperature well becomes filled with increasing time and that the amount of heat filling the well matches exactly that lost by the surrounding material. The rate of heat flowing into the well across the interface at x=1 equals-

$$\dot{q} = kA(T_H - T_C)\frac{\partial \tau}{\partial x}\Big|_{x=1} = \frac{kA(T_H - T_C)}{\sqrt{\pi}}\left\{\frac{1+\exp(\frac{-4}{\Delta^2})}{\Delta}\right\}$$

This result shows that the heat transfer rate becomes infinite as $\Delta \to 0$. Thus if we replace the well of temperature T_C rapidly and periodically in the sense that Δ remains small, a lot of heat will be transferred to the well. This is essentially what happens in our earlier found thermal pumping process where the well temperature is replaced periodically by fluid elements at different temperature.

In view of the above results it would appear that an even more efficient heat exchanger could be designed which would be quite superior to any steady-state devices presently used involving constant velocity counter-flows. It involves periodically removing the entire fluid present in the exchanger stack and repeating the process with a new batch of cold and hot fluid. The heat transfer device would look something like the following-

HEAT EXCHANGER SCHEMATIC
(Top View)



The heat exchanger stack consists of multiple channels of alternating hot and cold fluid separated by thin metal plates. It is also possible to replace the hot fluid channels by plates with an internal heat source such as micro-chips. Either approach should work. This type of exchanger closely resembles one discussed in one of our earlier patents (see US Patent #4976311 "An Oscillatory Liquid-Liquid Heat Exchanger".