# SPEED LIMITS ON TRAINS NEGOTIATING CURVES

The recent derailment of an Amtrack train outside of Philadelphia leading to the death of eight people and injuries to many more has again called peoples attention to the need for better speed control for trains entering curves , especially those curves which remain unbanked. The Amtrack train derailed while moving a little over 100mph into known dangerous curve marked for a top speed of 50mph. Although this crash was not as bad as a 1943 crash just north of Philadelhia in which 75 people perished do to the freezing of a wheel bearing, the recent crash was definitely avoidable and was clearly due to negligence on the part of the train's chief engineer. We want here to discuss the forces leading to train derailment when rounding sharp curves with the hope that future train engineers become better acquainted with the magnitude of the centrifugal forces created by a fast moving train entering a curve and the long time it takes to slow down a fast moving train even with all emergency brakes functioning..

Our starting point will be the following schematic-

SCHEMATIC OF THE PHILADELPHIA BOUND AMTRACK DERAILMENT
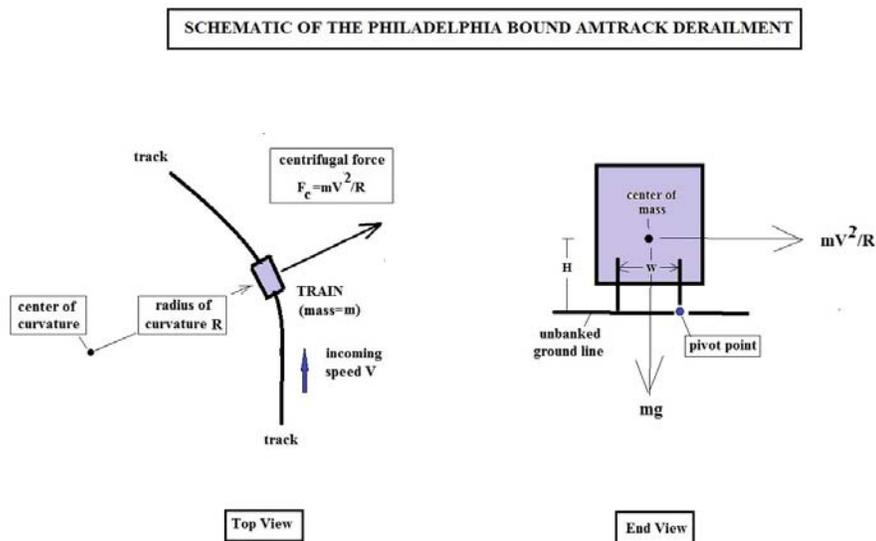
The left part of this picture gives a top view of a single passenger car entering a curve of radius R at V mph. The centrifugal force created at the sharpest part of the curve will be-

$$F_{centrifugal} = \frac{mV^2}{R}$$

The right side of the picture shows a rear view of the passenger car weighing mg pounds, with m its mass and g the acceleration of gravity. For the train to derail it is necessary that the moment about the pivot point shown is larger clockwise than counterclockwise. That is-

$$\frac{HmV^2}{R} > \frac{mgw}{2}$$

This result is independent of the mass and says the passenger car will derail if-

$$V^2 > \frac{gwR}{2H}$$

If one now takes w=56.5" representing the standard gauge, H=4ft, and R=300 ft, we get a derailment when-

$$V>sqrt[32(56.5/12)(300)/8]=75.166ft/sec=51.249mph$$

This number seems a little low and stems from the fact that there are retarding forces when multiple railway cars are hooked together which will safely allow somewhat higher speeds. Nevertheless it does show that when one enters a curve at double the recommended speed the centrifugal force goes up by a factor of four. To counter the tipping tendency one would need to widen the track gauge and lower the center of gravity. Of course there is another method to prevent derailment at high speed and that is to bank the track as they do in auto racetracks. We will come back to this point later.

At the moment let us see how many feet it would take to slow a train of ten Amtrack cars and a locomotive moving at 100mph to the lower speed of 50 mph. Each loaded passenger car weighs about 110 thousand pounds and the engine about 200 thousand pounds. This gives a total weight of the train at W= 1.3 million pounds. So the kinetic energy carried at 100mph is-

$$E = \frac{1}{2}mV^2 = \frac{WV^2}{2g} = 4.369x10^8 \, ft-lb$$

Slamming on the emergency brakes so that the wheels lock and wheel surface and track partially melt will produce a coefficient of friction of about μ= 0.1 or a braking force of –

$$F_{brake} = \mu W = 1.3x10^5 \, lbs$$

By Newton's second law, this produces a deceleration of-

$$a = -\frac{gF_{brake}}{W} = -\frac{32(1.3x10^5)}{1.3x10^6} = -3.2\,ft/\sec^2$$

So to go from 100mph to 50 mph will take a distance of-

$$S = \frac{(V_1^2 - V_2^2)}{2a} = \frac{7500(2.16)}{6.4} = 2531\,ft$$

That is, it will take a little less than one-half mile to accomplish this. The recent Philadelphia train derailment showed the chief engineer was able to reduce the train speed by only some 5mph, meaning that he did not react until he was some=

$$S = V\Delta V/a = 338\ ft$$

from the curve. There is no way the derailment could have been prevented given the speed of a little over 100 mpg just a few hundred feet ahead of the curve.

Let us next get back to the curve geometry and show how banking would allow much higher speeds than the 50mph limit imposed by Amtrack for this curve. Consider banking the curve by θ degrees. This produces the geometry shown in the following schematic-

TRAIN ON A BANKED TRACK

$\psi = \pi/2 - \arctan(w/2H) - \theta$

$\tan\varphi = w/(2H)$
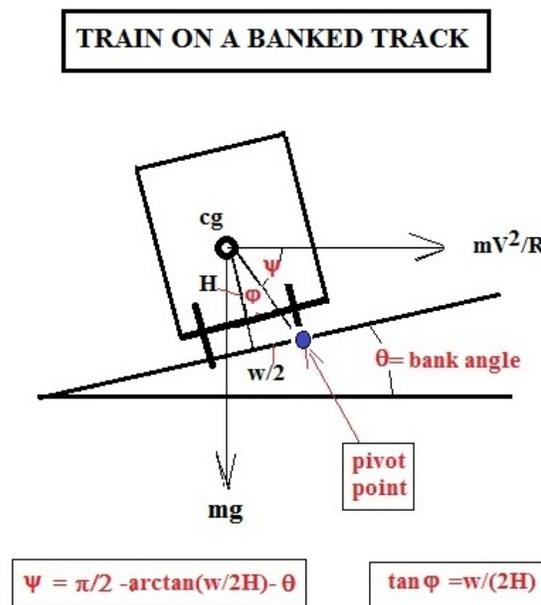
This time the clockwise moment about the pivot point is-

$$\frac{mV^2}{R}\sqrt{H^2+(w/2)^2}\sin\psi$$

and the counter-clockwise moment equals-

$$mg\sqrt{H^2+(w/2)^2}\cos\psi$$

Thus derailment will occur if-

$$\frac{V^2}{gR}\tan(\psi)\geq1$$

But we know from the geometry that-

tan(ψ)=tan[π/2-arctan(w/2H)-θ]

Hence, by application of the tan(A+B) formula several times, we have-

$$\tan(\psi)=\frac{1-\dfrac{w}{2H}\tan(\theta)}{1+\dfrac{w}{2h}\tan(\theta)}$$

So the derailment condition becomes-

$$\frac{V^2}{gR}\geq\frac{1+\dfrac{w}{2H}\tan(\theta)}{1-\dfrac{w}{2H}\tan(\theta)}$$

Note that here we have that the bank angle θ is a function of just two non-dimensional quantities-

$$\alpha=\frac{V^2}{gR}\qquad and\qquad \beta=\frac{w}{2H}$$

For an Amtrack train β is fixed to a value near 0.588 and α is confined to values less than one for zero banking. With banking larger speeds V are allowed. For example a curve banked at 20 degrees has tan(θ)=0.3639, so derailment will not occur until-

$$\alpha > \frac{1+0.588(0.3639)}{1-0.588(0.3639)} = 1.544$$

So the derailment speed will be sqrt(1.54)=1.24 times higher. Thus there is a definite improvement in the allowed maximum speed but it comes at the expense of the extra cost needed to construct a banked railway track and the fact that not all trains will have the same speed in entering the curve. A slow moving freight train entering such a curve could cause cargo to shift around inside the compartments and thereby possibly affect train stability. It is probably best to minimize the banking of railway tracks used for multiple purposes such as passenger traffic and freight movement. The very high speed passenger trains existing in the world, such as the Shinkansen in Japan , the Grande Vitesse in France and the China Star in China can all move at sustained speeds of more than 200 miles per hour on existing conventional tracks. Their tracks have clearly been designed to allow for such speeds. Curves are minimized as much as possible. Research on magnetically levitated trains (Maglevs) is also progressing at a rapid rate so that the day of 400mph trains is not far off provided that the economics of rail passenger traffic improves relative to airline competition and new straight line tracks (above or below ground) between major cities become available.

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