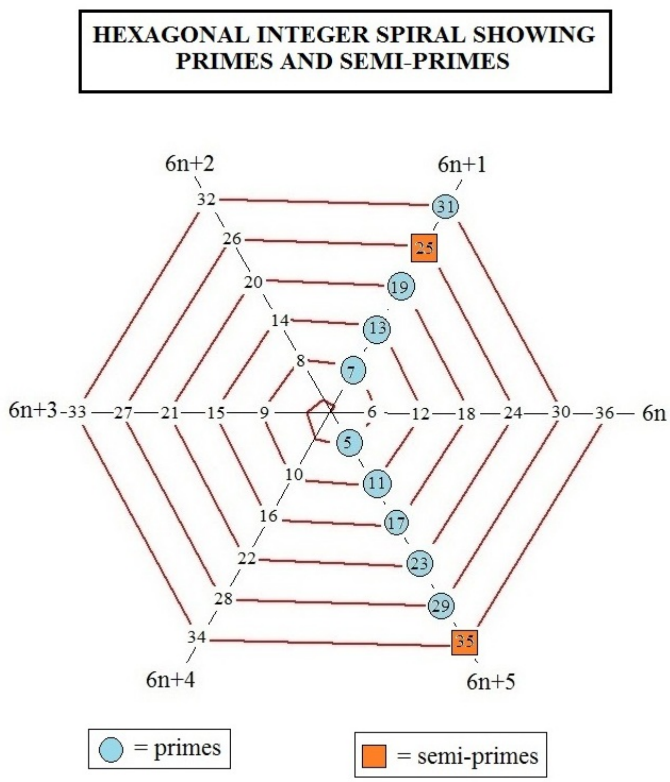
# CUMMULATIVE SUM OF TWIN PRIMES AS A FUNCTION OF N

Recently I watched a 1996 movie staring Jeff Bridges as a math professor at Columbia University. There is a funny scene in the movie were he(Bridges) points out to his rather unattractive but smart date (Barbra Streisand) that he is working on proving the correctness of the Twin-Prime Conjecture. The Conjecture states that there are an infinite number of twin-primes. This fact seems obvious but no one has yet succeeded in showing it to be correct. We will make an attempt here to show that the Conjecture is indeed valid.

Our starting point is the definition of a twin-prime. A twin prime represents essentially any two primes p and q which differ from each other by two. Thus [5,7] and [11,13] are examples. With the aide of our earlier discovered hexagonal integer spiral we can get an excellent visualization of all twin primes as shown-




From the graph one has a twin prime for q>p , only when

$$q=6n+1 \quad \text{and} \quad p=6n-1$$

are both primes. We consider only primes greater than three. Thus at n=1 we have q=6+1 and p=6-1 to form the twin-prime [p,q]=[5,7]. If n=10 we have q=60+1 and p=60-1 to get the twin prime [59,61]. Not all ns will work as the diagram shows for example for n=4 we have q=24+1=25 which is not a prime. It is now a simple matter to find all twin primes up through any integer value of N=6n using the simple one line MAPLE program-

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for n from 1 to N do {n,6n,isprime(6n+1),isprime(6n-1}od;
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A twin prime occurs whenever the output reads {n,6n,true,true}. All other answers are discarded. We are here particularly interested in finding the number of twin primes in any pre-chosen range of N=6n.To prove the twin-prime conjecture we now need to determine the cumulative sum S of all twin primes out to N=6n. This sum is easiest to determine by evaluating S(N) in one-hundred unit chunks of n. Taking [5,7] as the first twin prime, we find a total of 25 in the range of n going from 1 to 99. These correspond to-

n={1,2,3,5,7,10,12,17,18,23,25,30,32,38,40,45,47,52,58,70,72,77,87,95}

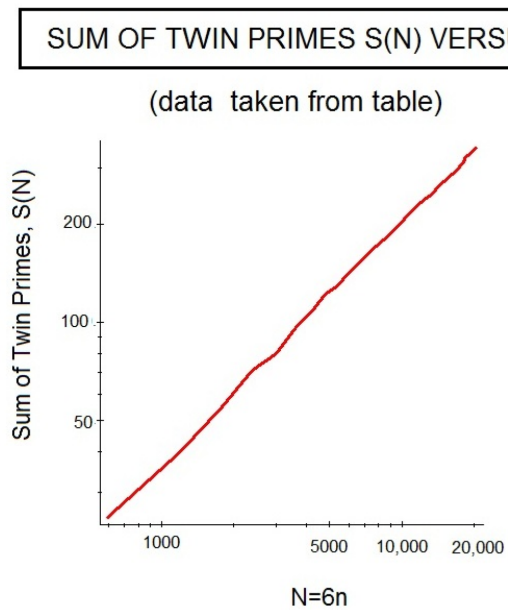
So, for example, n=72 corresponds to the twin prime [p,q]=[6(72)-1,6(72)+1]=[431,433]
Note that we do not consider [3,5] in our twin prime number count since p =3 does not fit our format that p=6n-1.

Continuing on to the second one hundred chunk we find 15 twin primes in the range 100≤n≤199. So the cumulative sum of twin primes to n=199 or N=1194 is S(N)=25+15=40. For the third hundred case we have S(N)=55. The following table gives N versus S(N) out to N=20394-

| n | N=6n | S(n) | n | N=6n | S(n) |
|---|---|---|---|---|---|
| 99 | 594 | 25 | 1699 | 10194 | 209 |
| 199 | 1194 | 40 | 1799 | 10794 | 217 |
| 299 | 1794 | 55 | 1899 | 11394 | 226 |
| 399 | 2394 | 71 | 1999 | 11994 | 234 |
| 499 | 2994 | 80 | 2099 | 12594 | 242 |
| 599 | 3594 | 96 | 2199 | 13194 | 247 |
| 699 | 4194 | 108 | 2299 | 13794 | 255 |
| 799 | 4794 | 122 | 2399 | 14394 | 265 |
| 899 | 5394 | 130 | 2499 | 14994 | 271 |
| 999 | 5994 | 142 | 2599 | 15594 | 277 |
| 1099 | 6594 | 151 | 2699 | 16194 | 287 |
| 1199 | 7194 | 162 | 2799 | 16794 | 293 |
| 1299 | 7794 | 172 | 2899 | 17394 | 301 |
| 1399 | 8394 | 180 | 2999 | 17994 | 314 |
| 1499 | 8994 | 188 | 3099 | 18594 | 324 |
| 1599 | 9594 | 198 | 3199 | 19194 | 329 |
| 1699 | 10194 | 209 | 3299 | 19794 | 337 |
| 1799 | 10794 | 217 | 3399 | 20394 | 345 |

What is most surprising about this list is that the number of twin primes in any 100 range of n for N>1000 or larger N remain at a finite value between 5 and 11. In none of the n chunks did a twin prime fail to appear.

A loglog point plot of this data produces the following graph-



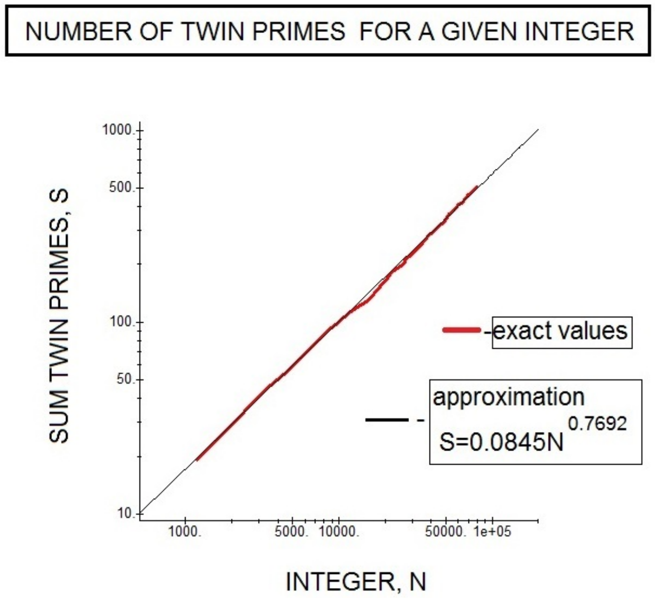

We see that the sum of the twin-primes up to N=6n follow a nearly linear relation of the form-

$$\ln(S)=a\ \ln(N)+b$$

, where a and b are to be determined by matching things at two points along the curve. We take these two points directly from the table. They have coordinates [N,S]=[1194,40] and [16794,293]. Solving for a and b we get the power law-

$$S(N)=0.19248\ N^{0.7532}$$

We represent this formula as a black line in a loglog plot and compare it with the exact result shown as a red line. Here is the combination graph-




The agreement is excellent. It allows us to give estimates for any value of N. Thus if N equals one million we have the approximation S=6332 and if N equals one billion then S equals approximately 1156606. The most important thing is that as N approaches infinity so does S(N). But if S(∞)=infinity then there must be an infinite number of twin-primes. Hence we have proof that the Twin-Prime Conjecture is correct!

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September 16, 2018
Gainesville, Florida
My 82nd Birthday