6.7 Hollow thin-wall torsion members

- Multiply connected sections are more complicated
- No shear stresses over holes, so stress function has zero slope

*Membrane for hollow torsion member.*
Thin-wall sections

- since $\tau = \frac{\partial \phi}{\partial n}$ where $n$ is normal to a membrane contour curve $z = \text{constant}$. Hence,

$$\tau = \left( \frac{2G\theta S}{p} \right) \frac{\partial z}{\partial n} = \left( \frac{2G\theta S}{p} \right) \tan \alpha$$

$$\tau = \frac{1}{c} \tan \alpha \approx \frac{1}{c} \sin \alpha$$

$$T = 2A\phi_1 = \frac{2Az_1}{c} = 2Aq = 2A\tau t$$

What is wrong with using the symbol $A$ for enclosed area?

Membrane for thin-wall hollow torsion member.
Torsional constant

• Z-equilibrium of soap film

\[ \sum F_z = pA - \int S \sin \alpha \, dl = 0 \]

\[ \frac{1}{A} \int \tau \, dl = \frac{p}{cS} = 2G\theta \]

• With \( q = \tau t \) being constant, it is instructive to write it in the form,

\[ \theta = \frac{1}{2GA} \int \tau \, dl = \frac{q}{2GA} \oint \frac{dl}{t} \]

• Altogether

\[ T = G"J"\theta \quad \quad "J" = \frac{4 A^2}{\oint \frac{dl}{t}} \]
Example

- Extruded aluminum tubing with a rectangular cross section has a torque loading of 24 kip-in. Determine the shearing stresses in each of the four walls with a) uniform wall thickness 0.160 in. b) uniform wall thickness of 0.120 in. on AB and AC & 0.2 in. on CD and BD

- Solution: First we calculate the shear stress

\[
A = (3.84 \text{ in.})(2.34 \text{ in.}) = 8.986 \text{ in}^2
\]

\[
q = \frac{T}{2A} = \frac{24 \text{ kip-in.}}{2(8.986 \text{ in}^2)} = 1.335 \frac{\text{kip}}{\text{in.}}
\]

\[
\tau = \frac{q}{t} = \frac{1.335 \text{ kip/in.}}{0.160 \text{ in.}} = 8.34 \text{ ksi}
\]
Example  Continued…

b) With variable thickness

\[
\tau_{AB} = \tau_{AC} = \frac{1.335 \text{ kip/in.}}{0.120 \text{ in.}} \implies \tau_{AB} = \tau_{AC} = 11.13 \text{ ksi.}
\]

\[
\tau_{CD} = \tau_{BD} = \frac{1.335 \text{ kip/in.}}{0.2 \text{ in.}} \implies \tau_{CD} = \tau_{BD} = 6.68 \text{ ksi.}
\]
Reading assignment

Sections 6.10: Why are there residual stresses in a shaft that is first loaded so that part or all of it yield and then unloaded? In a tension bar unloading leaves behind only strains, not stresses.

Source: www.library.veryhelpful.co.uk/ Page11.htm