A cylindrical pressure vessel of radius 2m is to be designed for pressure of 0.8 MPa (about eight atmospheres). If the pressure vessel were to be designed from aluminum with a yield stress of 500 MPa, then it would be designed by the hoop stress $\sigma_{\text{hoop}} = \frac{pr}{t} = 1.6 \times 10^6 / t$, so that the required thickness would be 3.2 mm. You are to design the skin from graphite-epoxy with the following properties (as given in Example 6.3.2): $X_t = X_c = 1500$ MPa, $Y_t = 40$ MPa, $Y_c = 246$ MPa, $S = 68$ MPa, $E_1 = 181$ GPa, $E_2 = 10.3$ GPa, $G_{12} = 7.17$ GPa, $\nu_{12} = 0.28$, $\alpha_1 = 0.02 \times 10^{-6} /^\circ C$, $\alpha_2 = 22.5 \times 10^{-6} /^\circ C$, $t_{\text{ply}} = 0.125$ mm. To calculate the thermal stresses, use the cure temperature as 180 $^\circ C$ and operating temperature of 10$^\circ C$.

1. Use the Hoffman criterion to design the lightest balanced and symmetric laminate that can carry safely the loads. Define the axial direction as the x direction, and then you expect a laminate that will have average ply orientation larger than 45$^\circ$, because the average axial stress is half of the average hoop stress. For this part, allow the number of plies to be a continuous design variable that can take non-integer values. Try first with an angle-ply laminate $(\pm \theta)_n$, so that you will have two design variables, and then with two angles, when you will have four design variables. Once you obtained a solution, round up the number of plies. When you have two angles, you can also try rounding one up and one down, as long as the total thickness is not reduced by the rounding.

2. Because the laminate thickness has to be an integer multiple of the ply thickness, the laminate will end up thicker than needed, and the strains could be lower than the allowables. Use this to change the design to have the highest possible shear modulus. This means, that after you find the minimum thickness in step 1, you design a laminate with this thickness that will not fail and has the lowest Poisson’s ratio.

3. Re-design the laminate to have only $0^\circ, \pm 45^\circ, 90^\circ$ plies. In addition, you cannot have any ply direction account for less than 10% or more than 50% of the thickness.

Due 3/22.