Problem 1

Apply the translation theorem to find the Laplace transforms of the functions.

\[ f(t) = t^4 e^{\pi t} \]

The translation theorem states that \( L \{ e^{at} f(t) \} = F(s - a) \).

For this problem, \( f(t) = t^4 \) and \( a = \pi \).

Therefore \( L \{ e^{\pi t} t^4 \} = \frac{4!}{(s - \pi)^5} = \frac{24}{(s - \pi)^5} \)
Problem 3

Apply the translation theorem to find the Laplace transforms of the functions.

\[ f(t) = e^{-2t} \sin 3\pi t \]

The translation theorem states that \( L \{ e^{at} f(t) \} = F(s - a) \).

For this problem, \( f(t) = \sin 3\pi t \) and \( a = -2 \).

Therefore \( L \{ e^{-2t} \sin 3\pi t \} = \frac{3\pi}{(s + 2)^2 + 9\pi^2} \)
Problem 7

Apply the translation theorem to find the inverse Laplace transforms of the functions.

\[ F(s) = \frac{1}{s^2 + 4s + 4} \]

\( F(s) \) can be rewritten as \( F(s) = \frac{1}{(s + 2)^2} \). This transform corresponds to an inverse of the form \( f(t) = t \). However we have \( s + 2 \) instead of \( s \). So \( a = -2 \) in the translation theorem.

Therefore \( f(t) = te^{-2t} \)
Problem 9

Apply the translation theorem to find the inverse Laplace transforms of the functions.

\[ F(s) = \frac{3s + 5}{s^2 - 6s + 25} \]

\[ F(s) \text{ can be rewritten as } F(s) = \frac{3(s - 3) + 5 + 9}{(s - 3)^2 + 16} = \frac{3(s - 3)}{(s - 3)^2 + 16} + \frac{14}{(s - 3)^2 + 16}. \]

This transform corresponds to an inverse of the form \( f(t) = \cos t + \sin t \). However we have \( s - 3 \) instead of \( s \). So \( a = 3 \) in the translation theorem.

Therefore \( f(t) = e^{3t}(3 \cos 4t + (7/2) \sin 4t) \).