,

ALL ABOUT SPIRALS

A spiral can be described as any 2D continuous curve where the radial distance r from the origin equals a specified function of the angle θ. Mathematically it has the unique derivative given by-

$$dr=(df/d\theta)d\theta \text{ with } r=0 \text{ at } \theta=0$$

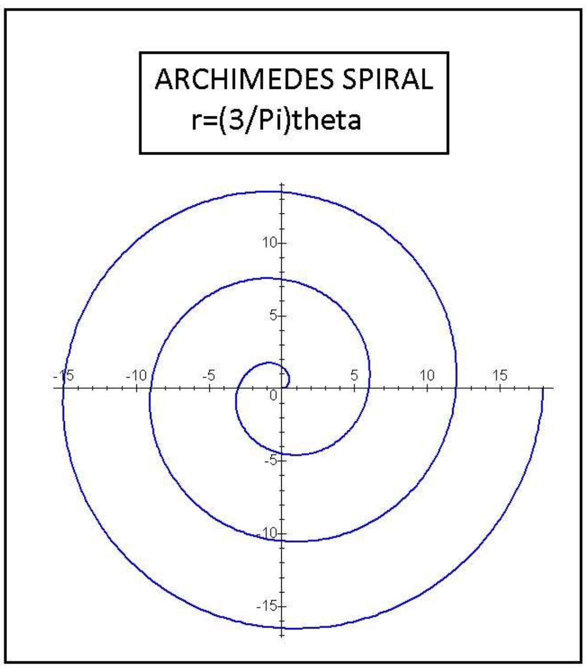
On integrating one finds-

$$r=\int_{t=0}^{\theta}\left(\frac{df(t)}{dt}\right)dt$$

This solution yields two simple forms. The first occurs when df/dt=α. We get-

$$r=\alpha(\theta)$$

This simplest of continuous spiral figures is just the Archimedes Spiral discovered by him over two thousand years ago. Setting the constant to α=3/π , we have the plot-

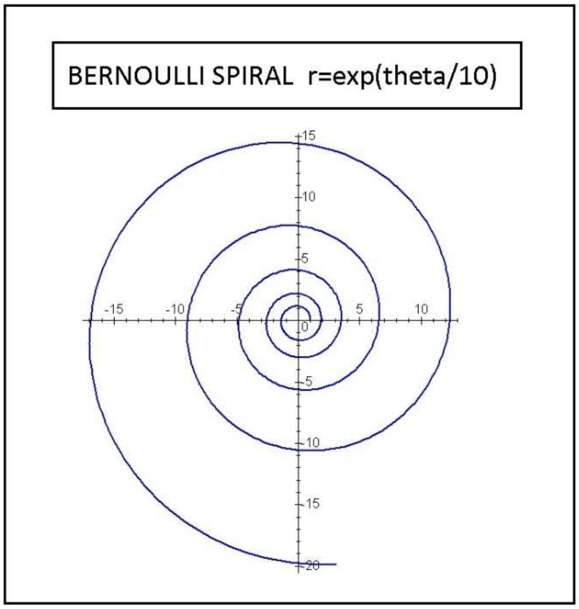


Note that the distance between turns of the spiral remain the same.

A second simple spiral is found when df/dθ=βf, where f=exp($\beta\theta$). If we take r=1 at θ=0, it produces-

$$r=\exp(\beta\theta) \qquad \text{or the equivalent} \qquad \ln(r)=\beta\theta$$

It is known as the Logarithmic Spiral or Bernoulli's Spiral. Here is its graph when β=1/10-



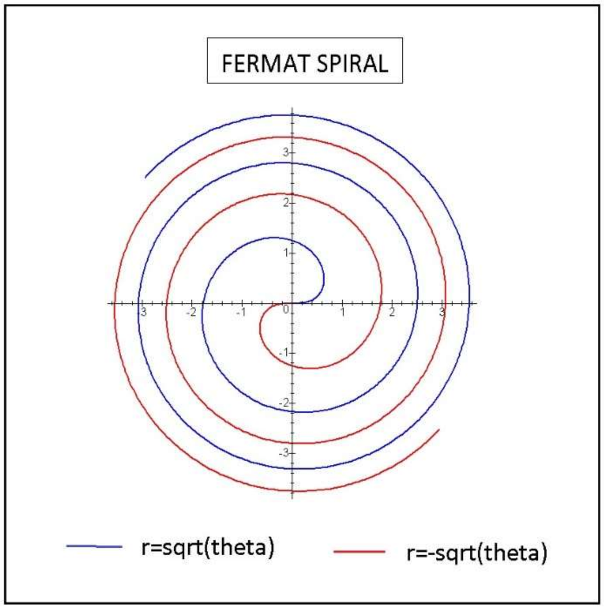
A property of the Bernouli Spiral is that the angle between any radial line and the tangent to the spiral remains a constant. J. Bernoulli was so intrigued by this spiral that he had a copy placed on his tombstone in Basel Switzerland. Here I am pointing to it back in 2000 -

Aug.2000 visit to Jacob BERNOULLI'S grave in Basel, Switzerland. I am pointing to the engraved Logarithmic Spiral ln(r)=A+B*theta to which Bernoulli ascribed mystical properties.

One can construct numerous other spirals by simply changing the form of df(θ)/dθ and picking a starting point $r(\theta_0)=r_0$. So if df/dθ=α/[2sqrt(θ)] and r=0 when θ=0, we get the spiral–

r=αsqrt(θ)

It looks as follows-

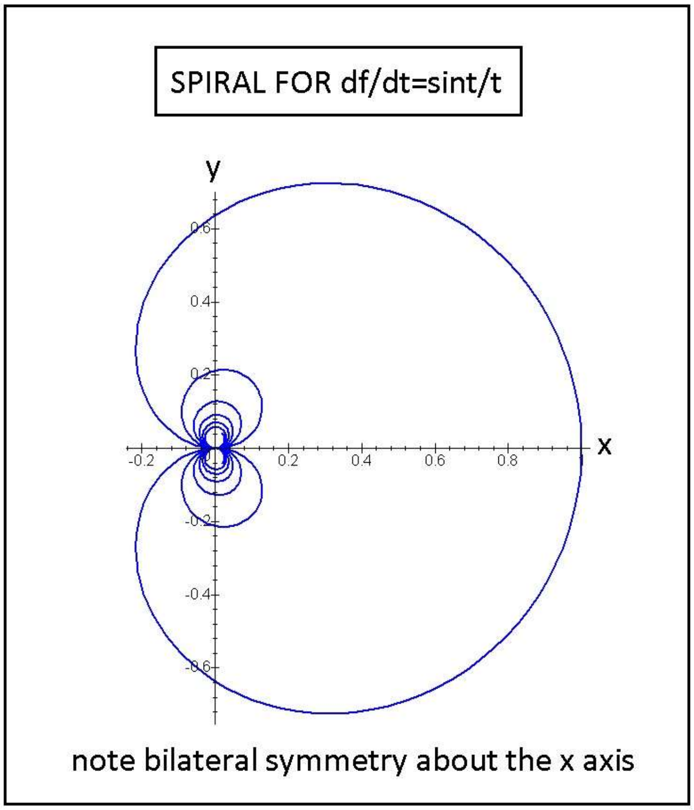


This figure is known as the Fermat Spiral. It has two branches, blue for α>0 and red for α<0.

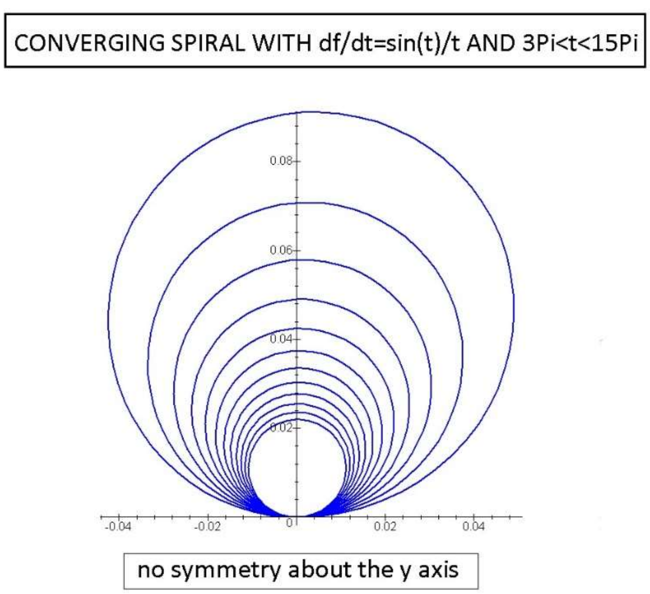
Next we look at the spiral –

r=int[sin(t)/t,t=-θ..θ]

It has a bilateral structure looking as follows-



We can also magnify that portion of the last solution between 3π<t<15π to get the following inward spiraling figure-



As θ heads toward infinity the spiral goes toward r=0.

Another interesting looking spiral is generated by-

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plot([t*BesselJ(0,t),t-Pi/4,t=6..50],coords=polar,thickness=2,color=blue,
scaling=constrained,numpoints=800);
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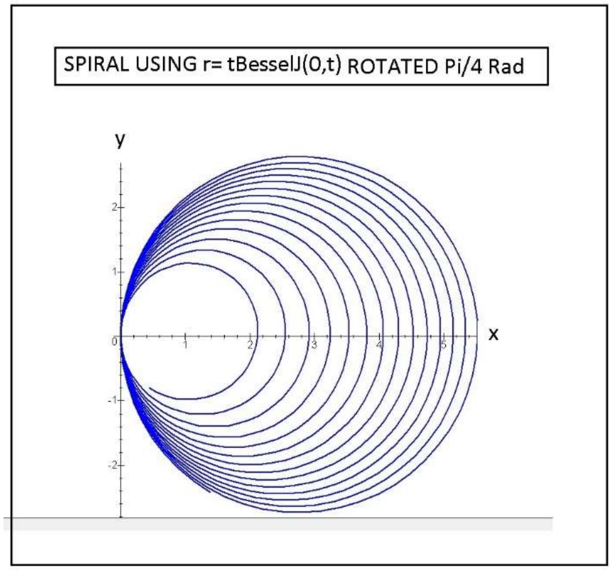
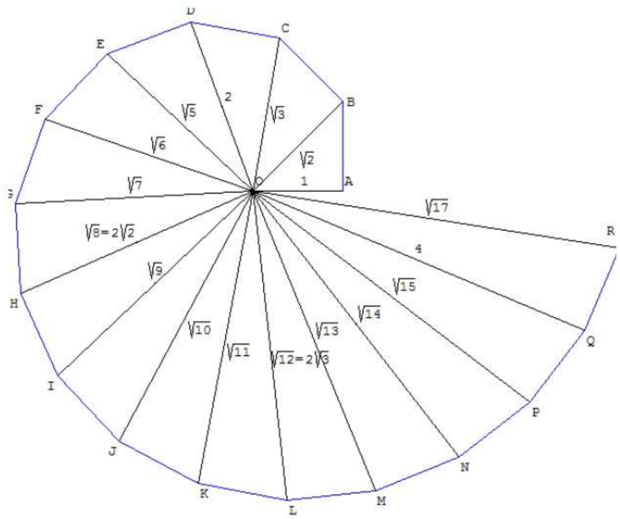
Here is the graph-



One can generate an infinite number of other continuous spirals such as the Cornu Spiral, the Euler Spiral, and spirals based on Fresnel integrals. We will skip these particular constructions and head instead directly over to spirals with jumps in their derivatives. One of the earliest known of these is the Spiral of Cyrene first produced by Theodorus of Cyrene in the 5th century BC. It consists of piled up right triangles whose hypotenuse increases by one unit in each step. Here is its image-

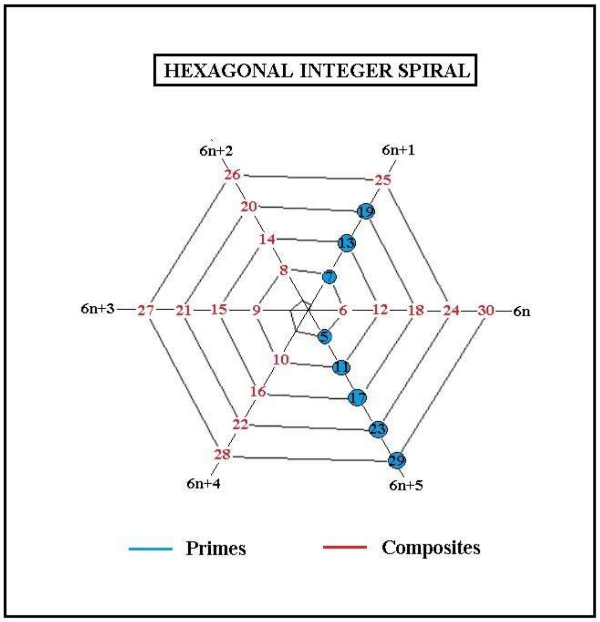


SPIRAL OF CYRENE

Another spiral , with this time a jump in its curvature between neigboring squares , is the Fibonacci Spiral as shown on the 49cent US postage stamp-

Here the pattern is created by breaking a rectangle up into its golden ratio proportions and then using a compass to draw quarter circles in each square.

A third spiral showing discontinuities at its connecting lines is the hexagonal Integer spiral discovered by us about a decade ago and used to locate prime numbers in the z plane. Here is its image-



Finally let us take a look at some spirals as they appear in nature. The following give some of the better known ones.

We start with a typical hurricane spiral-

HURRICANE KATRINA-August 2005
showing counterclockwise rotating spiral

Note the clear eye of the hurricane. The maximum wind speeds occur at the eye wall and then drop off from there with increasing radial distance following approximately the formula $p+\rho V^2/2=\text{const}$. That is, the lower the pressure at the eye wall the higher the wind speed.

Next we look at the spiral structure of the Messier 101 Galaxy in Ursa Major-

THE MESSIER 101 SPIRAL GALAXY

It lies at about 22 million light years from earth and has a diameter of approximately 170,000 light years.

Next we look at the Nautilus Sea Shell. When sliced it reveals a chambered structure resembling the logarithmic spiral. Here is an image taken from Earth Sky-

Cut-Open Nautilus Sea Shell and its Resemblance to the Bernoulli Spiral

Lastly, consider the following Sea-Shell. Looking at it from the top shows a Bernoulli (or Logarithmic) Spiral Structure-

SEA-SHELL EXIBITING LOGARITHMIC SPIRAL STRUCTURE

The structure is believed due to repeated growth of the preceding pattern. The more turns the older the shell.

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