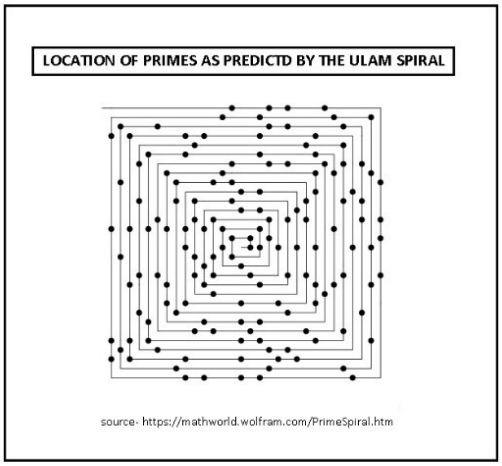
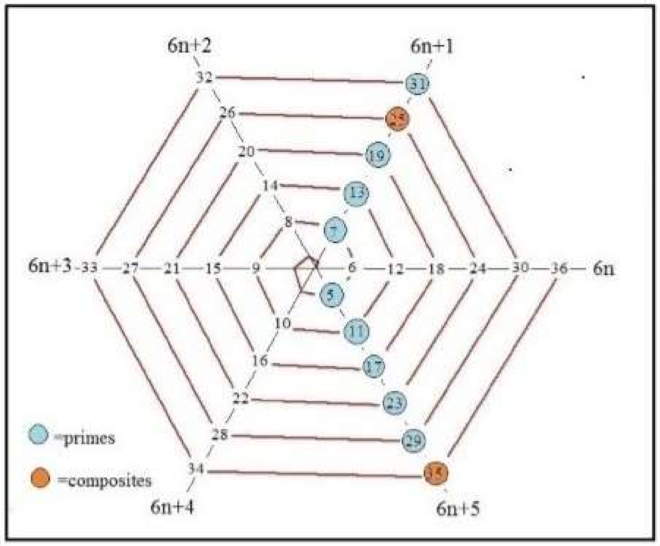
ON THE GENERATION OF INTEGER SPIRALS

It is well known that one can plot all positive integers in the form of spirals. Two of the better known forms involve the Ulam Spiral dating back to 1963 and our own Hexagonal Integer Spiral first found back in 2013 (see google reference " morphing Ulam") . The typical Ulam Spiral has prime numbers given in the following quasi-irregular fashion-.



For years mathematicians tried to come up with an explanation for this pattern without success. It was not until about a decade ago , when we first showed that all primes greater greater than three must have the form $6n\pm1$, that a much simpler spiral pattern for integers emerged. Our idea was to place the integers N at points $[r,\theta]=[N,N\pi/3]$ along a Archimedes Spiral $r=(3/\pi)\theta$ and then connect neighboring vertexes by straight lines. The resultant effort yielded the following considerably simplified spiral pattern where all primes five or greater lie along just one of two radial lines $6n\pm1$-

HEXAGONAL INTEGER SPIRAL



Note that 6n+5 is equivalent to 6n-1 in this representation. We have called this pattern a hexagonal integer spiral for obvious reasons. The integers are found at the vertexes of the resultant spiral with each turn of the spiral yielding six additional integers. Thus numbers along the same radial line differ by multiples of six. The gaps in primes lying along radial lines 6n±1 will often represent semi-primes especially if the numberfraction f=[σ-N-1]/N is a number just slightly above zero. Thus the orange number 35 on the graph equals 5x7 and has an f value of 0.3428. The testing whether or not a number N is a prime is to first carry out a mod(6) operation. Only possible primes have N mod(6) equal to either 1 or 5. For example-

1812830360399 mod(6)=5

says that N lies along the radial line 6n-1 and could thus possibly is a prime. Using our prime test f(N)=(sigma(N)-N-1)/N=0 confirms that 1812830360399 is indeed a prime. On the other hand 4698127359mod(6)=3 so 4698127359 must be a composite.

There are several additional observations which can be made about the hexagonal integer spiral. For example, a twin prime has two neighboring primes p and q differing by two units. The only way this can be is to have the average (p+q)/2 be a factor of six while p =6n-1 and q=6n+1 with p<q. From the above graph we can read off that four twin primes are [5,7],[11,13],[17,19],and [29,31].

Another fact one can confirm with the hexagonal integer spiral is the Goldbach conjecture that any even number can be expressed as the sum of two primes. This observation follows by noting that each of the following prime number combinations produce even numbers-

(6n+1)+(6m+1)=6(n+m)+2→ $even(even) + even$=even
(6n-1)+(6m+1)=6(n+m)+0→ $even(even) = even$
(6n-1)+(6m-1)=6(n+m)+0→even(even)=-even

A last observation concerning the hexagonal integer spiral is that the sigma function of number theory can be read directly off of the graph by noting that if N is a prime(blue circle) then-

$$\sigma(N)=N+1$$

This means that the sigma of the semi-prime 35 equals 6 x 8=48. Also more generally we have that for any semi-prime N=pq it is true that –

$$\sigma(N)=N+(p+q)+1$$

Thus σ(66277)=66277+(191+347)+1=66816.

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