GENERATING LARGE PRIME NUMBERS USING PRODUCTS OF IRRATIONAL FUNCTIONS

About a decade ago, while studying the factoring of large semi-primes, we came up with a new technique for generating large prime numbers p based on the infinite series expansions of a group of irrational functions evaluated at fixed points. Unlike primes obtained by using a collection of random numbers, the present approach not only generates such primes more quickly but also makes it possible to transmit such primes in an extremely compact form via the internet. It is our purpose here to discuss this approach in more detail.

Let us begin by finding a ten digit long prime number p based on the series expansion -

C:= sin(1)= .8414709848

Dropping the decimal point produces the ten digit number-

F:= 8414709848

Next carrying out the one line search program-

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for d from -10 to 10 do {d,isprime(F+d)}od;
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produces a prime when d=3 or -9. Picking the value of d closest to zero leads to the ten digit long prime-

p=8414709848+3=8414709851

Here we have (p)mod(6)=5 so that the number lies along the radial line 6n-1 of a hexagonal integer spiral. The only other possibility for a prime would be p mod(6)=1. Note that this prime can be transmitted via the internet to any friendly receiver via the simple condensed version-

V=10*(F+d)

Here the ten refers to the number of digits in the desired prime while d is the smallest departure fron d-0 to make F+d a prime. All that is required is that the friendly receiver knows C which can be made quite complicated , subject to rapid change, and transmissible via encrypted form.

We try next to find a larger 30 digit long prime starting with the combination-

C:=exp(2)*erf(1)/cos(3)=0.402052710379165959484565945640

Removing the decimal point then produces the 30 digit long number-

F:= 402052710379165959484565945640

Next carrying out the search-

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for d from -50 to 50 do{d,isprime(F)}od;
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produces d=-31.This means we get the 30 digit long prime-

p=402052710379165959484565945640-31=402052710379165959484565945609 .

This prime has p mod(6)=5 and hence is also of the form 6n-1.

As a third example , we consider generating a 90 digit length prime starting with the combination
C:=(1+sqrt(5)*Pi^(1.5)/exp(3)=
0.89713747696533391853627802131376273816309745592729851359057047976883574588062148303 7243653.

Removing the decimal point then yields the ninety digit long number-

F:=
897137476965333918536278021313762738163097455927298513590570479768835745880621483037243653

Carrying out a search with this F produces the requirement that d=134. Hence we have the 90 digit prime-
p:=F+134=897137476965333918536278021313762738163097455927298513590570479768835745880621483037243787

This last prime has pmod(6)=1 meaning it lies along the radial line 6n+1 in a hexagonal integer spiral.

As shown by the above examples evaluated at fixed points, large primes can be rapidly produced. These should be of value in connection with public key cryptography. The fact that such large prime can be packed into small packages means that unbreakable public keys involving semi-primes N=pq can be rapidly constructed and electronically transmitted. Only the friendly receivers will know what the value of C being used is. The only slight disadvantage of the present approach is that the absolute value of d will generally increase with increasing size of p. This will not be a problem for primes of 100 digit length or so for which d will remain in the hundreds.

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