FURTHER PROPERTIES OF PRIMES, SEMI-PRIMES, AND DOUBLE-PRIMES

We have shown in several recent articles starting about 2013 that all primes greater than p=3 have the form 6n±1, with n being any positive integer. This is a necessary but not sufficient condition for making a number prime but as we will see below it does restrict the primes to this form and no other. Here is a quick list of all 6n±1 numbers from n=1 through n=16 showing their prime or composite values-

Number n	6n±1	Prime or Composite
1	5, 7	p,p
2	11,13	p,p
3	17,19	p,p
4	23,25	p,c=5x5
5	29,31	p,p
6	35,37	c=5x7,p
7	41,43	p,p
8	47,49	p,c=7x7
9	53,55	p,c=5x11
10	59,61	p,p
11	65,67	c=5x13,p
12	71,73	p,p
13	77,79	c=7x11,p
14	83,85	p,c=5x17
15	89,91	p,c=7x13
16	95,97	c=5x19,p

All the prime numbers between 2 and 97 are-

2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97.

There are exactly 25 of these. Subtracting out the first two, leaves us with 23 primes which matches exactly the number of primes (p) listed in the above table. We have carried out the prime calculation for numbers through N=4000 and find no exception to the rule that –

All primes above p=3 have the form 6n±1.
That is p mod(6) is either +1 or +5

Take the case of the prime p=35689137903245699 . It reads p=6(5948189650540950)-1 and has p mod(6)=5
The number N= 45682309167543167543 cannot be a prime but must be a composite (C) since N mod(6)=3.

You will notice from the above table that if $6n\pm1$ is not a prime then the number is composite and made up of powers of products of primes. Thus-

$$N=77=6(13)-1=7x11 \quad \text{and} \quad N=91=6(15)+1=7x13$$

Both of these numbers are semi-primes. Since they represent the product of two primes, it must also be true that semi-primes have the form $6k\pm1$, with k an integer. We can state that all semi-primes, composed of primes p and q greater than 3, must have the form-

$$N=6k\pm1=p \text{ x } q$$

This fact gives one a way to factor large semi-primes into its p and q components. Let us demonstrate. Take the semi-prime N=7991 where N mod(6)=5. This means-

$$N=6(1332)-1=(6n+1)(6m-1)$$

Solving for n we get-

$$n=\frac{(1332-m)}{(6m-1)}$$

We need an integer solution to this Diophantine equation. The simplest way to obtain it is to work out (1332-m)mod(6m-1) by varying m until this operation yields zero. A zero first occurs at m=22 and forces n to be 10. Thus the semi-prime factors as-

$$N=7991=[6(10)+1][6(22)-1]=61 \text{ x } 131$$

One has some guidance as to what range n and m to use in the search noting that n and m lie on opposite sides of sqrt(N)/6 which here equals about 15.

A very interesting result (stemming from the fact that all primes above 3 and all semi-primes which have both its components p and q greater than 3) is these numbers all have the form $6s\pm1$, where s is an integer. This fact allows one to construct a hexagonal integer spiral defined in polar coordinates$[r,\theta]$ by-

$$r=N \quad \text{and} \quad \theta=\pi N/3$$

In computer language this spiral is defined by-

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listplot([seq([N,N*Pi/3],N=0..36)],coords=polar,color=red,
symbol=circle,axes=none,thickness=2,scaling=constrained):
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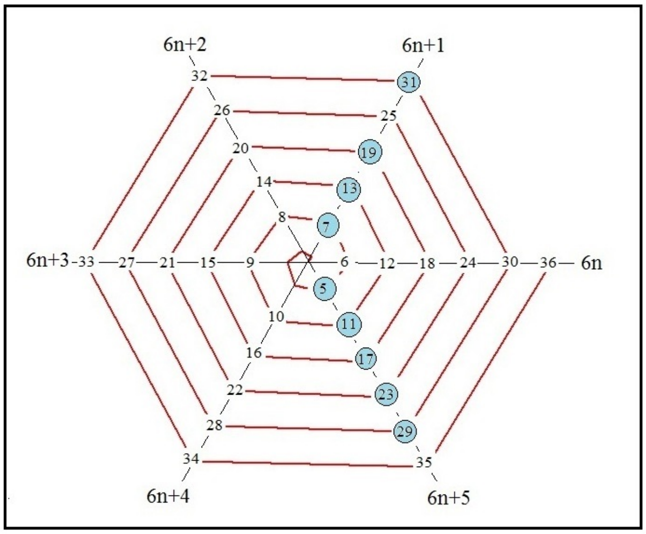
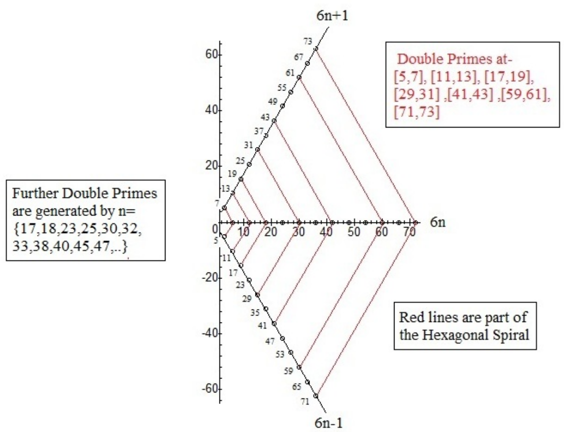
It produces the following hexagonal integer spiral-



In the graph we have also drawn in six radial lines corresponding to N mod(6)=0 through N mod(6)=5. You will note that the primes shown all lie exclusively along the 6n+1 and 6n-1 radial lines. It is interesting that, despite of extensive efforts by the mathematics community devoted to drawing elaborate graphs of the classic Ulam Spiral, no one has, prior to our 2013 article(see http://www2.mae.ufl.edu/~uhk/MORPHING-ULAM.pdf) noted that all primes above p=3 can be made to fall along just these two radial lines. The primes are circled in light blue in the above graph. Note the gaps at n=25=5x5 and 35=5x7. These represent semi-primes. A semi-prime of the form 6k-1 will have its two components have the form p=6n+1 and q=6m-1. A semi-prime of the form 6k+1 will have p=6n+1 and q=6m+1 or p=6n-1 and q=6m-1. We also note that some of the gaps along the 6n±1 lines can be filled with non-semi-primes such as N=26273=13x43x47. However, pure semi-primes within the required size restriction will always have N mod(6) equal to one or five. The spacing between primes along either 6n+1 or 6n-1 will always be equal to integer factors of six. Thus the prime p=6(3)-1=17 differs from the prime 6(5)-1=29 by 2x6=12. One also encounters double-primes (sometimescalled twin orimes) which are characterized by having p-q=+2 or -2. These can directly read off of the above graph. The graph shows the double-primes 5-7, 11-13, 17-19, and 29-31. It shows that they exist only when both 6n +1 and 6n-1 are primes. Here is a graph showing a few more of the smaller double-primes-

LOCATION OF THE SMALLER DOUBLE PRIMES



As n gets large these double primes become less frequent. A couple of large double-primes occur at N+1 and N-1 for N=6(357413)=2144478 and N= 6(3874561022712)= 23247366616272.

A ninety digit long prime we found many years ago by combining the digits appearing as products of certain irrational constants such as π,e, and sqrt(2) is-

p=312025858078207203648521755410552436986027465111950912853
165341074749716615733412809020767

If one asks what is the value of a mod(6) and isprime(p) operation on this number, my computer gives the answer-

p mod(6)=1 , isprime(p)=true

That is, we have a prime number sitting along the 6n+1 radial line at the p/6 th turn of the hexagonal spiral.

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