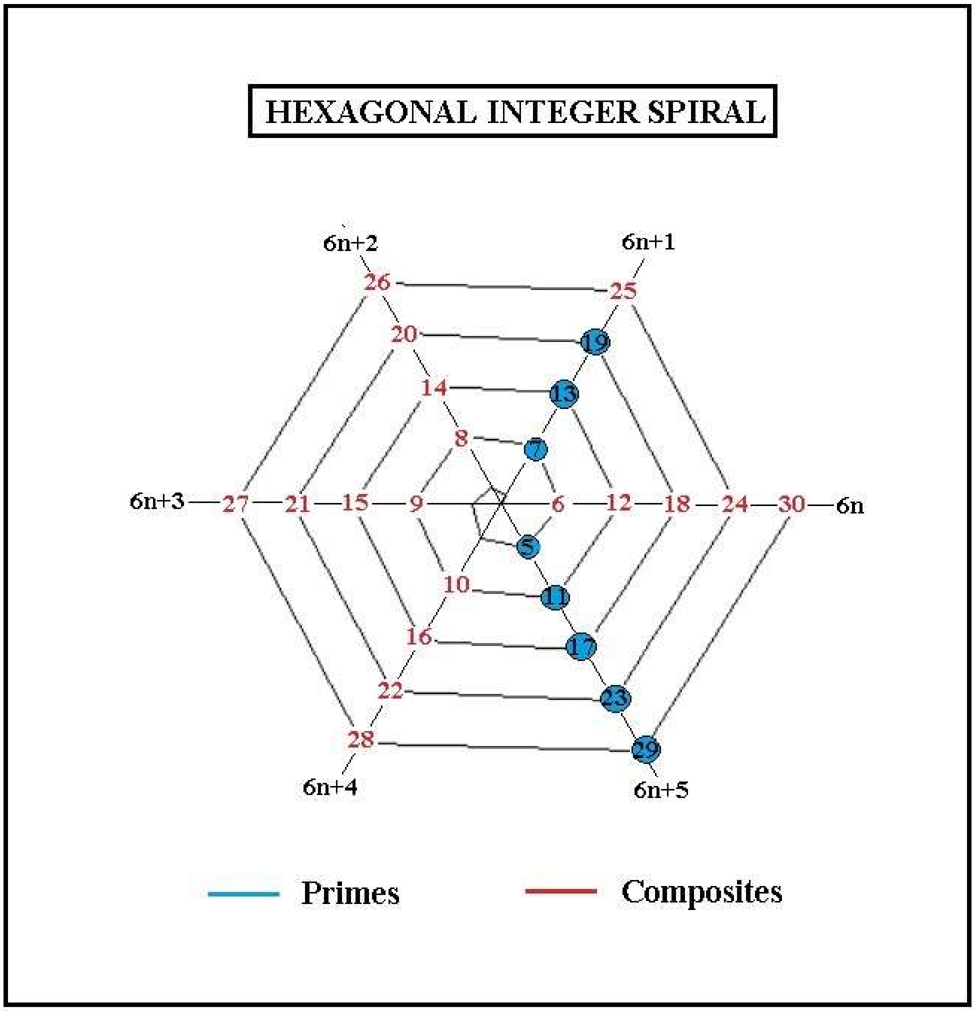
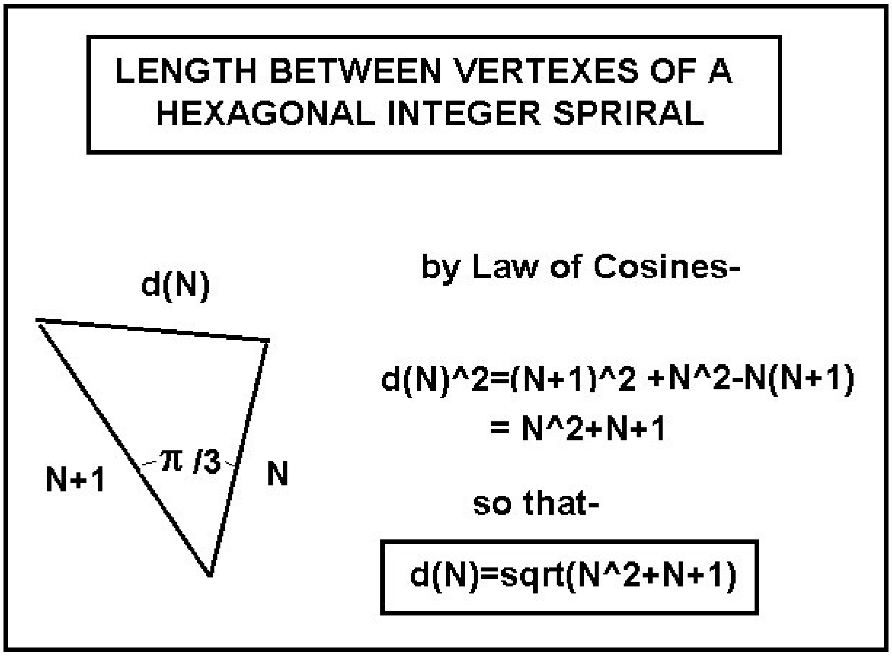
ON A NEW SET OF PRIMES GENERATED BY A SIMPLE QUADRATIC FORMULA

About a decade ago we came up with a new way to express primes appearing in the Ulam Spiral in a much simpler way in which all prime numbers five or larger lie strictly at the intersection of the vertexes of a hexagonal spiral and two radial lines 6n±1. The resultant picture is the following-



We have been able to use this new integer projection to rapidly locate twin primes, to show that the number fraction f(N)=[sigma(N)-N-1)/N vanishes when N is a prime, and that N mod6)=1 or 5 is a requirement for N to be a prime. In the present note we wish to introduce a new subgroup of primes derived directly from the above hexagonal integer graph and discuss some of its more important properties.

We start by looking at two neighboring integers along the spiral located in polar coordinates as $[r,\theta]=[N,\pi N/3]$ and $[N+1, \pi(N+1)/3]$. A magnified picture of these two points and the origin produces the following not quite equilateral triangle-



By the Law of Cosines one has that the square of the distance between the two neighboring integers equals-

$$d(N)^2=(N+1)^2+N^2-2N(N+1)\cos(\pi/3)=N^2+N+1$$

Here are the first few distances -

N	d(N)
1	sqrt(3)
2	sqrt(7)
3	sqrt(13)
4	sqrt(21)
5	sqrt(31)

You will note that most of the numbers inside the square roots in this able are primes of the form $p=N(N+1)+1$. Here $p \bmod(6)=1$, indicating that these primes

lie along the radial line 6n+1.To get primes to lie along 6n-1 require that we also look at p=N(N+1)-1, where p mod(6)=5 (or –1). This observation suggests the existence of <u>a new sub-group of primes</u> defined by the quadratic formula-

$$K(N)=N(N+1)\pm 1$$

We call these the K primes. A table for these , very simple to generate primes, follows-

N	N(N+1)+1	N(N+1)-1
1	3	-
2	7	5
3	13	11
4	-	19
5	31	29
6	43	41
7	-	-
8	73	71
9	-	89
10	-	109
11	131	-
12	157	-
13	-	181
14	211	-
15	241	239
16	-	271
17	307	-
18	-	-
19	-	379
20	421	419
21	463	461
22	-	-
23	-	-
24	601	599
25	-	-

Here the dashes locate where K(N) will not be a prime. A count shows that just twenty percent of the numbers in the above table are actually K primes .This percentage will decrease as N gets larger. Although not yet proven, it seems reasonable from the above that the total number of K primes will be infinite. The primes in column two have the property that N mod(6)=1 and the Ns in column three all satisfy N mod(6)=5. The fastest way to find a K prime in the neighborhood of any large number N_0 is to use the search program-

for N from (N_0-b) to (N_0+b) do ({N,isprime(K(N))})od

, where b represents a small integer compared to N_0. A search in the narrow range where b=10 about the seven digit number N_0=1234567 yields-

p=1234573

Its mod(6) value is one, so that this prime lies along the 6n+1 radial line at the 205762nd turn of the hexagonal integer spiral.

A really large K prime can be generated by looking at the number-

N= 1415926535897932384626433832795028841972

This number is derived from π after dropping the 3 and then going out 40 digits. Doing a computer search in the range No-60 to No+60 produces several primes. These correspond to b=[-53,45,51] . They read , in that order,-

N= 1415926535897932384626433832795028841919

= 1415926535897932384626433832795028842017

= 1415926535897932384626433832795028842023

Clearly this last result indicates that the K primes can be generated by a simple quadratic formula in N . The density of these primes decreases as N increases. A mod(6) number operation tells whether they lie along the 6n+1 or 6n-1 radial line. In the above case the mod operation yields [+5,1,1]. Note that because of modular operations periodicity, the mod(6) operation 5 is equivalent to -1. The value of f(N)=[sigma(N)-N-1]/N is zero , as expected, for each of the above three cases. Note that these K(N) primes have a much larger density than those given by the Mersenne Primes 2^N-1.

U.H.Kurzweg
April 11, 2020
Gainesville, Florida