PRIMES AND SUPER-COMPOSITES

Over the last decade or so we have been studying primes, semi-primes, twin primes and super-composites in great detail. Much of the new material discovered by us can be found by carrying out the Google search-

MATHFUNC-Topic of Interest- u.h.kurzweg

So, for example,-

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brings up-

https://mae.ufl.edu/~uhk/TWIN-PRIMES.pdf

and-

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yields-

https://mae.ufl.edu/~uhk/NUMBER-FRACTION-REVISITED.pdf

It is our purpose here to look at some additional features on two of our most important discoveries, namely, number fraction and the hexagonal integer spiral. The Number Fraction is defined as-

$$f(N)=[\sigma(N)-N-1]/N$$

Here the numerator represents the sum of all factors of N excluding N and 1.The σ(N)is the sigma function of Number Theory. N in the denominator prevents N from growing too rapidly with increasing N. What is most interesting about f(N) is that it will vanish whenever N is a prime. Thus f(147846917)=0. We can also look at the additional point function-

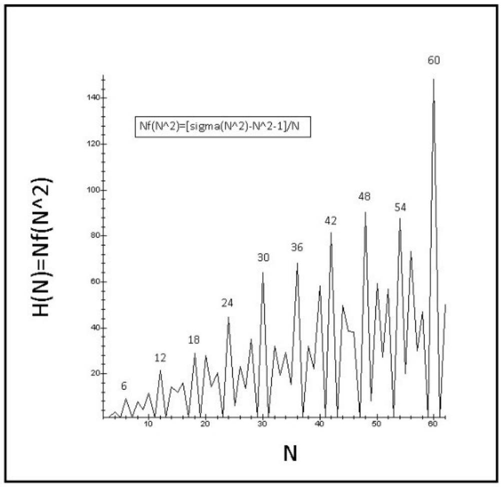
$$H(N)=Nf(N^2)=[\sigma(N^2)-N^2-1]/N$$

One finds that prime numbers are found whenever f(N)=0 or H(N)=1. Rewriting f(N) , we also have the identity -

$$\sigma(N)=1+N[1+f(N)]$$

Since the sigma function is given out to about 40 digits for N in advanved math programs such as MAPLE or MATHEMATICA, the same will be true for f(N). Whenever N is a prime we have σ(N)=1+N.

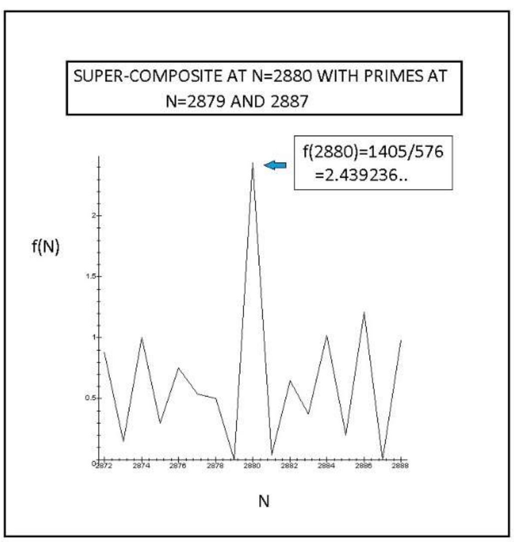
A plot of H(N) versus N follows-



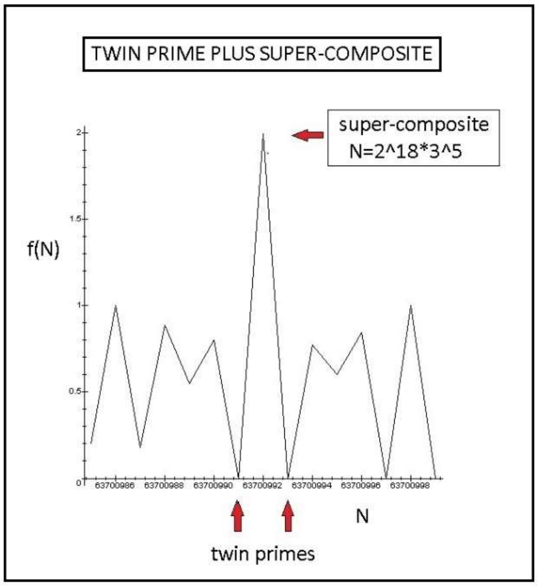
Whenever Nf(N^2)=1 we get the primes 5,7,11,13,17,19,23,29,31,37,41,43,47,53,57,59, and 61.Those special primes differing from each other by two are referred to as <u>Twin Primes</u>. On the graph they are [5,7],[11,13],[17,19],[29,31],[41,43], and [59,61]. Note that their average value equals 6n,where n=1,2,3,5,7,10.We'll come back later in this article to show why this should be so. Both f(N) and H(N) have local maxima which we call Super-Composites. The local maxima of H(N) or f(N) are designated as <u>Super-Composites</u>. They are found for Ns of the form-

N=(2^a)(3^b)(5^c)(7^d)…

, whith a>b>c>d . So, for instance, a plot of f(N) in the neighborhood of N=2^6*3^2*5=2880 looks as follows-

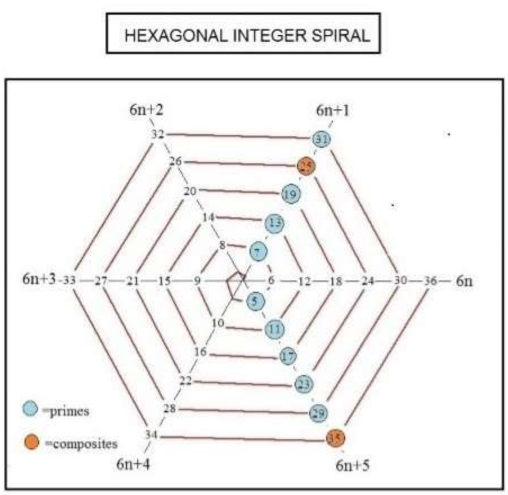


We indeed haves a super-composite at N=2880 but only one prime in the immedeate vicinity of N. This means there is no twin prime possible which has 2880 as its mean value. However we can change this by looking at N=2^18*3^5=63700992. This has f(N)=1.995879… with both f(N-1) and f(N+1) vanishing. Such variations in the Number Fraction produce a Twin Prime. Here is a graph in the neighborhood of N-



The reason for the near symmetry about f(N) is not clear at the moment but may become of importance in future studies.

To better understand Twin Primes we can make use of our earlier discovered hexagonal integer spiral. This patten looks as follows-



We see here that all primes above three have the form N=6n±1. This is a necessary but not suffienet conditon because some of the points along these two radial lines are composites such as 25 and 35. Twin Primes have the property that their average value is 6n, that they differ from each other by two, and must have f(6n+1)=f(6n-1)=0. Reading from the graph the first few Twin Primes read-

[5,7] , [11,13] , [17,19] , [29,31]

We note that all primes fall along just two radial lines 6n+1 and 6n-1. Because there are also a few composites along these lines, one can make the restricted statement that-

A necessary but not suffiicient condition that N is a prime is that N=6n±1 , with integer n=1 or higher.

Semi-primes are defined as N=pq with p and q equal primes. Since p and q must lie along the radial lines 6n±1, so does N. We have for such semi-primes the governing equations-

N=pq and p+q=Nf(N)

Solving for p and q, we get-

$$[p,q]=S\pm sqrt[S^2-N]$$

Here S=(p+q)/2=[sigma(N)-N-1]/2. So the semi-prime N=5662379 has sigma(N)=5667720 and S=2670. So the primes p and q read-

[p,q]=2670±sqrt(1466521)=[1459,3881] .

One can factor any semi-prime N=pq as long as sigma(N) is given by one's PC. So the key to breaking public keys in cryptography is to find an improved methods for calculating σ(N) rapidly.

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